

1. Find the area of the region(s) enclosed by the line $y = 4x$ and the curve $y = x^3 + 3x^2$.

Intersection points:

$$4x = x^3 + 3x^2$$

$$0 = x^3 + 3x^2 - 4x$$

$$0 = x(x^2 + 3x - 4)$$

$$0 = x(x+4)(x-1)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x=0 & x=-4 & x=1 \end{array}$$

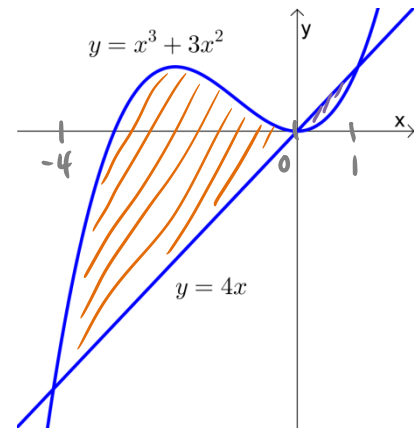
$$\int_{-4}^0 [x^3 + 3x^2 - 4x] dx$$

$$= \left[\frac{x^4}{4} + x^3 - 2x^2 \right]_{-4}^0$$

$$= 0 - \left(\frac{256}{4} - 64 - 32 \right) = 32$$

$$\begin{aligned} & \int_0^1 [4x - (x^3 + 3x^2)] dx \\ &= \left[2x^2 - \frac{x^4}{4} - x^3 \right]_0^1 \\ &= \left(2 - \frac{1}{4} - 1 \right) - 0 \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{Total area} &= 32 + \frac{3}{4} \\ &= \boxed{32\frac{3}{4}} \quad (32.75) \end{aligned}$$



2. Find the area of the region enclosed by the curves $x - y^2 = 0$ and $x + 2y^2 = 3$. (Hint: Integrate in y direction.)

$$x = y^2 \quad x = -2y^2 + 3$$

Intersection points:

$$y^2 = -2y^2 + 3$$

$$3y^2 = 3$$

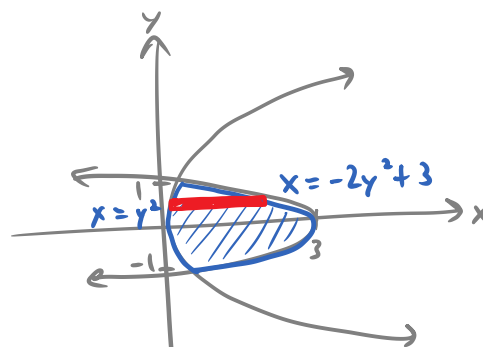
$$y^2 = 1$$

$$y = \pm 1$$

$$\text{Area} = \int_{-1}^1 [(-2y^2 + 3) - (y^2)] dy$$

$$= \int_{-1}^1 (-3y^2 + 3) dy$$

$$= 2 \int_0^1 (-3y^2 + 3) dy$$



$$= 2 \left[-y^3 + 3y \right]_0^1$$

$$= 2(-1 + 3)$$

$$= \boxed{4}$$