1. Express $\frac{x}{2x^2+1}$ as a power series and find its interval of convergence.

$$\frac{x}{2x^2+1} = x \cdot \frac{1}{1-(-2x^2)}$$

$$= x \cdot \sum_{n=0}^{\infty} (-2x^2)^n$$

$$= x \cdot \sum_{n=0}^{\infty} (-2)^n x^{2n}$$

$$= \boxed{\sum_{n=0}^{\infty} (-2)^n x^{2n+1}}$$

Interval of convergence:

$$|-2x^2| < 1$$

$$2x^2 < 1$$

$$x^2 < \frac{1}{2}$$

$$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

$$\boxed{\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}$$

2. Find a power series representation for $\tan^{-1} x$. Then plug in $x = 1$. (Caution: your brain might explode.)

Note: $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$$\int \frac{1}{1+x^2}\,dx = \int \frac{1}{1-(-x^2)}\,dx = \int \left(1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \dots\right)dx = \int \sum_{n=0}^{\infty} \underbrace{(-x^2)^n}_{(-1)^n x^{2n}}\,dx$$

$$\tan^{-1} x = \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right) + C = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} + C$$

Find C (plug in $x=0$):

$$\tan^{-1} 0 = \sum_{n=0}^{\infty} (-1)^n \frac{0^{2n+1}}{2n+1} + C$$

$$0 = C$$

$$\tan^{-1} x = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}}$$

$$\tan^{-1} 1 = \sum_{n=0}^{\infty} (-1)^n \frac{1^{2n+1}}{2n+1}$$

$$\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

A FORMULA FOR π!

Q: What has one head, one foot, and four legs?