

1. Find the radius and interval of convergence of  $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$ .

$$\left| \frac{(3x-2)^{n+1}}{n+1} \cdot \frac{n}{(3x-2)^n} \right| = \left| \frac{(3x-2)n}{n+1} \right| = \frac{n}{n+1} |3x-2| \rightarrow |3x-2| \text{ as } n \rightarrow \infty$$

By Ratio Test,  $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$  converges when  $|3x-2| < 1$

$$3|x - \frac{2}{3}| < 1$$

$$|x - \frac{2}{3}| < \frac{1}{3}$$

$$R = \frac{1}{3}$$

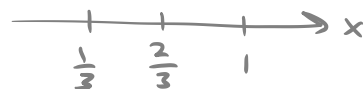
Test endpoints:

$$x = \frac{1}{3}: \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \leftarrow \text{Converges by AST}$$

$$x = 1: \sum_{n=1}^{\infty} \frac{1}{n} \leftarrow \text{diverges (harmonic series)}$$

Interval of convergence:

$$\frac{1}{3} \leq x < 1$$



2. Find the radius and interval of convergence of  $\sum_{n=1}^{\infty} (\ln n)x^n$ .

$$\left| \frac{\ln(n+1)x^{n+1}}{\ln(n)x^n} \right| = \frac{\ln(n+1)}{\ln(n)} |x| \rightarrow |x|$$

$$\lim_{n \rightarrow \infty} \frac{\ln(n+1)}{\ln(n)} = \lim_{n \rightarrow \infty} \frac{\ln(\frac{1}{n+1})}{\ln(\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{n}{n+1} = 1$$

By Ratio Test,  $\sum_{n=1}^{\infty} (\ln n)x^n$  converges when  $|x| < 1$

$$R = 1$$



Test endpoints:

$$x = -1: \sum_{n=1}^{\infty} (\ln n)(-1)^n \leftarrow \text{diverges by Test for Divergence}$$

$$x = 1: \sum_{n=1}^{\infty} (\ln n) \leftarrow \text{diverges by Test for Divergence}$$

Interval of convergence:

$$-1 < x < 1$$

3. Find the radius and interval of convergence of  $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$ .

$$\left| \frac{2^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n x^n} \right| = \left| \frac{\cancel{2} \cdot \cancel{2}^n \cdot x \cdot \cancel{x}^n}{(n+1) \cdot \cancel{n}!} \cdot \frac{\cancel{n}!}{\cancel{2}^n \cancel{x}^n} \right| = \left| \frac{2x}{n+1} \right| = \frac{2}{n+1} |x| \rightarrow 0$$

as  $n \rightarrow \infty$   
(for all  $x$ )

$$R = \infty$$

$$\text{Interval of convergence: } (-\infty, \infty)$$

Q: There is a word in the English language in which the first two letters signify a male, the first three letters signify a female, the first four signify a great man, and the whole word, a great woman. What is the word?