Alternating Series

Alternating Series Test (AST)

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + \cdots$$

converges if *all three* conditions are satisfied:

1. $b_n > 0$
2. $b_{n+1} \leq b_n$ for all $n \geq N$ (that is, the absolute value of the terms are eventually nonincreasing)
3. $b_n \to 0$ (that is, $\lim\limits_{n\to\infty} b_n = 0$)

Ex 1.

Does $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ converge or diverge?

Ex 2.

Does $\sum_{n=2}^{\infty}(-1)^n\frac{4}{(\ln n)^2}$ converge or diverge?

Ex 3.

Does $\sum_{n=1}^{\infty}(-1)^n \frac{3n}{4n-1}$ converge or diverge?

For an alternating series, how close is s_n to the sum of the infinite number of terms?

Alternating Series Estimation Theorem

Suppose we have an alternating series $s = \sum_{n=1}^{\infty}(-1)^{n-1}b_n$ where $b_n > 0$, $b_{n+1} \leq b_n$, and $b_n \to 0$.
Then, $|R_n| = |s - s_n| \leq b_{n+1}$

In other words, difference between s_n and the entire sum is less than or equal to the absolute value of the next term.

Ex 4.

Find the sum of the series $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}$ correct to 3 decimal places.

Practice

1. Does $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt[7]{n}}$ converge or diverge?

2. Does $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^2}{n^3+1}$ converge or diverge?

Challenge: Show by example that $\sum a_n b_n$ may diverge even if $\sum a_n$ and $\sum b_n$ both converge.

Q: What is harder to catch the faster you run?