1. Does $\sum_{n=8}^{\infty} \frac{1}{\sqrt[3]{n}-1}$ converge or diverge?

$$\frac{1}{\sqrt[3]{n}-1} \geq \frac{1}{\sqrt[3]{n}} \quad (\text{for } n \geq 8)$$

Since $\sum_{n=8}^{\infty} \frac{1}{\sqrt[3]{n}}$ diverges (p-series with $p=\frac{1}{3}$),

$\sum_{n=8}^{\infty} \frac{1}{\sqrt[3]{n}-1}$ also $\boxed{\text{diverges}}$ by comparison.

2. Does $\sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$ converge or diverge?

$$\frac{2^n}{3+4^n} \leq \frac{2^n}{4^n} = \left(\frac{2}{4}\right)^n = \left(\frac{1}{2}\right)^n \quad (\text{for } n \geq 1)$$

Since $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ converges (geometric series with $r=\frac{1}{2}$),

$\sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$ also $\boxed{\text{converges}}$ by comparison.

3. Does $\sum_{n=1}^{\infty} \frac{n+3}{n^4-n^3+2n}$ converge or diverge?

Think: $\frac{n+3}{n^4-n^3+2n} \sim \frac{n}{n^4} = \frac{1}{n^3}$

Case 1

$$\lim_{n\to\infty} \frac{\left(\frac{n+3}{n^4-n^3+2n}\right)}{\left(\frac{1}{n^3}\right)} = \lim_{n\to\infty} \frac{n^4+3n^3}{n^4-n^3+2n} = \lim_{n\to\infty} \frac{1+\frac{3}{n}}{1-\frac{1}{n}+\frac{2}{n^2}} = 1$$

By LCT, since $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges (p-series with $p=3$),

$\sum_{n=1}^{\infty} \frac{n+3}{n^4-n^3+2n}$ also $\boxed{\text{converges}}$.

Challenge: Does $\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$ converge or diverge?

Challenge: $\sum_{n=1}^{\infty} \frac{1}{1+2+\dots+n} = \sum_{n=1}^{\infty} \frac{1}{\left(\frac{n(n+1)}{2}\right)} = \sum_{n=1}^{\infty} \frac{2}{n^2+n}$

$\boxed{\text{Converges}}$ by comparison to $\sum_{n=1}^{\infty} \frac{2}{n^2}$

Q: What is the word that everybody always says wrong?