1. Write the first four terms of the sequence defined below. (This generates the **Fibonacci numbers**.)

$a_1 = 1, \quad a_2 = 1, \quad a_{n+1} = a_n + a_{n-1}$

$a_1 = \boxed{1}$

$a_2 = \boxed{1}$

$a_3 = a_2 + a_1 = 1 + 1 = \boxed{2}$

$a_4 = a_3 + a_2 = 2 + 1 = \boxed{3}$

2. Find a formula for the nth terms of the sequence $5, \frac{3}{2}, \frac{1}{3}, -\frac{1}{4}, -\frac{3}{5}, -\frac{5}{6}, -1, \ldots$

top: subtract 2

bot: add 1

$$\boxed{a_n = \frac{7-2n}{n}}$$

3. Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence.

a) $a_n = (-1)^n\left(1 - \frac{1}{n}\right)$

$\lim_{n\to\infty} (-1)^n\left(1 - \frac{1}{n}\right)$ does not exist (even terms approach 1, odd terms approach −1)

$\{a_n\}$ **diverges**

b) $a_n = 2 + (-1)^n$

$\lim_{n\to\infty} \left[2 + (-1)^n\right]$ does not exist (oscillates between 1 and 3)

$\{a_n\}$ **diverges**

c) $a_n = \frac{2n+1}{1-3\sqrt{n}}$

$$\lim_{n\to\infty} \frac{2n+1}{1-3\sqrt{n}} = \lim_{n\to\infty} \frac{2\sqrt{n} + \frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}} - 3} = -\infty$$

$\{a_n\}$ **diverges**

d) $a_n = \left(\frac{2}{n}\right)^{1/n}$

$\ln a_n = \ln\left(\frac{2}{n}\right)^{1/n} = \frac{1}{n}\ln\left(\frac{2}{n}\right) = \frac{\ln 2 - \ln n}{n}$

$\lim_{n\to\infty} \ln a_n = \lim_{n\to\infty} \frac{\ln 2 - \ln n}{n} = \lim_{n\to\infty} \frac{-1/n}{1} = 0$

↑ L'Hosp

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} e^{\ln a_n} = e^0 = \boxed{1}$ Converges

e) $a_n = \frac{1-\cos n}{n^2}$

$-1 \le \cos n \le 1$) $\cdot(-1)$

$-1 \le -\cos n \le 1$) $+1$

$0 \le 1 - \cos n \le 2$) $\div n^2$

$0 \le \frac{1-\cos n}{n^2} \le \frac{2}{n^2}$

Since $\lim_{n\to\infty} 0 = \lim_{n\to\infty} \frac{2}{n^2} = 0$,

we have $\lim_{n\to\infty} \frac{1-\cos n}{n^2} = \boxed{0}$ by the Squeeze Theorem.

converges

Q: What are the next two letters in this sequence: A E F H I K L M ?