

1. Find the Taylor series for $\frac{1}{x}$ at $x = 2$ using the definition of a Taylor series.

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f''(x) = \frac{2 \cdot 1}{x^3}$$

$$f'''(x) = \frac{-3 \cdot 2 \cdot 1}{x^4}$$

$$f^{(4)}(x) = \frac{4 \cdot 3 \cdot 2 \cdot 1}{x^5}$$

$$f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}} \quad \xrightarrow{x=2} \quad f^{(n)}(2) = \frac{(-1)^n n!}{2^{n+1}}$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!} (x-2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n \cancel{n!}}{\cancel{n!} \cdot 2^{n+1}} (x-2)^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-2)^n$$

(Note: If we didn't have to use the definition of a Taylor series, we could write:

$$\frac{1}{x} = \frac{1}{2+x-2} = \frac{1}{2\left(1 - \left(-\frac{x-2}{2}\right)\right)}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \left(-\frac{x-2}{2}\right)^n \quad \left. \begin{array}{l} \text{Geometric} \\ \text{Series} \end{array} \right\}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-2)^n$$

2. Find a Maclaurin series for $f(x) = x^2 \ln(1+x^3)$ by using $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$.

$$x^2 \ln(1+x^3)$$

$$= x^2 \cdot \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x^3)^n}{n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^{3n+2}$$

Q: When can you add two to eleven and get one as the correct answer?