

Test #3 Review Exercise Answers

1. $y' = Ae^x - 5Be^{-5x}$ and $y'' = Ae^x + 25Be^{-5x}$, so
 $y'' + 4y' - 5y = Ae^x + 25Be^{-5x} + 4(Ae^x - 5Be^{-5x}) - 5(Ae^x + Be^{-5x})$
 $= Ae^x + 4Ae^x - 5Ae^x + 25Be^{-5x} - 20Be^{-5x} - 5Be^{-5x}$
 $= 0$

2. $y' = te^{-t} - e^{-t}$ and $y'' = -te^{-t} + e^{-t} + e^{-t} = -te^{-t} + 2e^{-t}$, so
 $y'' + 2y' = -te^{-t} + 2e^{-t} + 2(te^{-t} - e^{-t}) = te^{-t}$.
 Also, $y(0) = -(0)e^{-0} + 6 = 6$, and $y'(0) = 0e^{-0} - e^{-0} = -1$.

- 3.
- a. $T > T_S$
 - b. $T < T_S$
 - c. $T = T_S$

- 4.
- a. $y = \sin\left(\frac{1}{2}\ln(x^2 + 1) + C\right)$
 - b. $y = -\ln(e^{-x} + C)$
 - c. $y = 2 - Ce^{-x^2/2}$

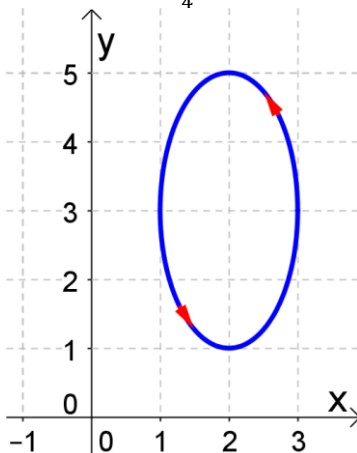
- 5.
- a. $-\frac{2}{2x^2+2x-1}$
 - b. $y = \pm \sqrt{e^{-2x}\left(-x^2 - x - \frac{1}{2}\right) + 1}$

6. $\frac{2}{3}x^2 + y^2 = C$

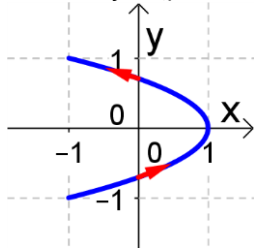
7. $y = \frac{C}{x}$

8. 7.7925% (the DE is $\frac{dA}{dt} = 2 - \frac{A}{10}$ and the solution to the DE is $A(t) = 20 - 12e^{-t/10}$, $A(10) = 15.585$ gallons of alcohol)

- 9.
- a. $(x - 2)^2 + \frac{(y-3)^2}{4} = 1$ (ellipse)



b. $x = 1 - 2y^2$ (parabola opening left)



10. $y - \frac{2}{3} = \frac{3}{2}(x - 3)$ (or $y = \frac{3}{2}x - \frac{23}{6}$)

11. $\frac{1}{(1-\sin t)^2}$ (Note: $\frac{dy}{dx} = \frac{\cos t}{1-\sin t}$)

12. $\frac{2}{27}(37\sqrt{37} - 10\sqrt{10})$ (Hint: do some algebra and then let $u = \sqrt{t}$ to solve integral)

13. $\frac{2\sqrt{2}}{5}\pi(2e^\pi + 1)$

14.

a. $\frac{9\pi}{2}$ (Setup: $4 \cdot \int_0^{\pi/2} \frac{1}{2}(3 \sin 2\theta)^2 d\theta$)

b. $18\sqrt{3} - 4\pi$ (Setup: $\int_{-\pi/3}^{\pi/3} \frac{1}{2}(4 + 4 \cos \theta)^2 d\theta - \int_{-\pi/3}^{\pi/3} \frac{1}{2}(6)^2 d\theta$)

c. $\frac{5\pi}{4}$ (Setup: $2 \cdot \left(\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}(1 + \sin \theta)^2 d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{2}(\sin \theta)^2 d\theta \right)$)

d. $\frac{5\pi}{4}$ (Setup: $2 \cdot \left(\int_0^{\frac{\pi}{3}} \frac{1}{2}(1 + \cos \theta)^2 d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(3 \cos \theta)^2 d\theta \right)$)

e. $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ (Setup: $2 \cdot \left(\int_0^{\frac{\pi}{6}} \frac{1}{2}(2 \sin \theta)^2 d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(1)^2 d\theta \right)$)

f. $4\pi - 6\sqrt{3}$ (Setup: $\int_{7\pi/6}^{11\pi/6} \frac{1}{2}(2 + 4 \sin \theta)^2 d\theta$)

15. $\frac{\sqrt{5}}{2}(e^4 - 1)$ (Setup: $\int_0^2 \sqrt{(e^{2\theta})^2 + (2e^{2\theta})^2} d\theta$)

16. 2π (Setup: $\int_0^{\pi\sqrt{2}} \sqrt{(\sqrt{1 + \sin 2\theta})^2 + \left(\frac{\cos 2\theta}{\sqrt{1 + \sin 2\theta}}\right)^2} d\theta$)

17. $-\sqrt{3}$

18. Horizontal: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$; Vertical: $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (Note: you'll have to take the limit as $\theta \rightarrow \frac{\pi}{2}$ of $\frac{dy}{dx}$ to determine what happens at $\frac{\pi}{2}$)

19.

a. converges to $-\frac{\pi}{2}$

b. converges to 0 (Hint: use Squeeze Theorem)

c. converges to $\sqrt{2}$

d. diverges

e. converges to 1

f. converges to 0

g. converges to 0

- h. converges to 0
- i. converges to 1
- j. converges to 0 (separate, then Squeeze Theorem)
- k. converges to 0 (geometric sequence)

20. 5

21. $1 + \sqrt{2}$

22.

- a. divergent (Limit Comparison Test with $\frac{1}{n}$)
- b. divergent (Test for Divergence)
- c. convergent (Alternating Series Test)
- d. convergent (Integral Test)
- e. converges to $\frac{64}{125}$ (geometric series with $a = \frac{64}{625}$ and $r = \frac{4}{5}$)
- f. converges to $-\frac{2}{33}$ (geometric series with $a = -\frac{2}{27}$ and $r = -\frac{2}{9}$)
- g. divergent (geometric series with $r = \frac{\pi}{3} > 1$)
- h. divergent (Test for Divergence)
- i. convergent (Comparison Test with $\frac{2}{n^{3/2}}$)
- j. convergent (Comparison Test with $\frac{5}{n^2}$)
- k. convergent (Alternating Series Test)
- l. convergent (Alternating Series Test)

23.

- a. $-\frac{\pi}{4}$
- b. $\frac{3}{2}$

24. $\frac{3979}{550}$

25.

- a. At most $\frac{1}{484}$ (≈ 0.0021)
- b. At least 112 terms
- c. 0.05184824862

26.

- a. At most $\frac{1}{\ln 6}$ (≈ 0.55811063)
- b. At least 485,165,196 terms (that is, $> e^{20}$)
- c. 2.1120790183 (Note that $s_6 \approx 1.5760745338$ represents the sum of the terms $n = 2$ through $n = 6$, which is the first five terms. Your integrals should give you $\frac{1}{\ln 7}$ and $\frac{1}{\ln 6}$.)

27. -0.2834 (Need to add first 4 terms to get this, so that $|s - s_4| < b_5 = \frac{1}{3^{5!}} = \frac{1}{29160} < 0.00005$.)28. 0.9721 (Need to add first 7 terms to get this, so that $|s - s_7| < b_8 = \frac{1}{8^5} = \frac{1}{32768} < 0.00005$.)