Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and hand them in at the test, you can earn up to 3% extra credit towards your test (depending on neatness and completeness)! It is important to understand that these review exercises are <u>not guaranteed to cover all</u> of the potential problems on the test. Please review the notes, practice problems, previous quizzes, and homework problems to fully prepare for the test.

1. Find the following integrals. If the integral diverges, write "diverges."

   a. $\int \frac{2x^3+3x-3}{x^4+2x^2+1}dx$

b. $\int \frac{-5x^2-12x-43}{(x-1)(x+3)(x^2+4)}\,dx$

c. $\int \frac{\sqrt{x}}{x^2+3x}\,dx$

d. $\int \frac{1}{\sqrt{\sqrt{x}+1}}\,dx$

e. $\int \frac{1}{1+e^{2x}}\,dx$

f. $\int x^3 e^{x^2}\,dx$

g. $\int_0^\infty xe^{-2x}\,dx$

h. $\int_{-\infty}^{\infty} \frac{dx}{x^2+9}$

i. $\int_{-\infty}^{0} \frac{xdx}{\sqrt{4-x}}$

j. $\int_{0}^{2} \frac{dx}{(x-1)^{1/3}}$

k. $\int_{1}^{3} \frac{xdx}{\sqrt{3-x}}$

2. Consider the integral $\int_1^3 x \ln x \, dx$.
   a. Use the Trapezoidal Rule with $n = 4$ steps to approximate the integral, and then estimate the error in the approximation.

   b. How large does n need to be to guarantee that the approximation from part (a) is accurate to within 0.00001?

   c. Use Simpson's Rule with $n = 4$ steps to approximate the integral, and then estimate the error in the approximation.

   d. How large does n need to be to guarantee that the approximation from part (c) is accurate to within 0.00001?

   e. Evaluate the integral directly.

3. Show that each of the following integrals either converge or diverge using the Comparison Test.

a. $\int_1^{\infty} \frac{x^2-x-3}{\sqrt{x^8+x^5+2}}dx$

b. $\int_2^{\infty} \frac{x^2+2}{(x-1)^3}dx$

c. $\int_3^{\infty} \frac{\sin^2 x+x+x^5}{\sqrt{x^{12}-5x^7-23x}}dx$

d. $\int_2^{\infty} \frac{\sqrt{x}-1}{x^3+3x^2}dx$

4. Find the length of each of the following curves.

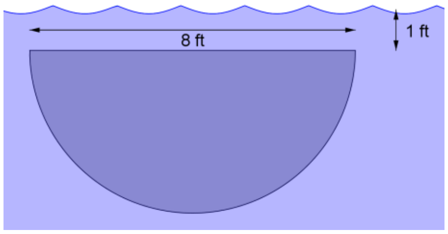
a. $y = \ln(\cos x)$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$

b. $y = \frac{2}{3}(x^2 + 1)^{3/2}$ from $x = 1$ to $x = 4$

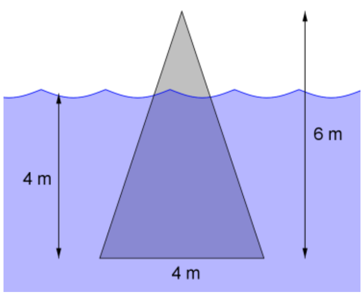
5. Find the area of the surface generated by revolving the curve $y = x^3$ from $x = 0$ to $x = 1$ about the x-axis.

6. Find the area of the surface generated by revolving the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = 1$ to $x = 3$ about the x-axis.

7. A semicircular vertical plate is submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.



8. A vertical plate in the shape of an isosceles triangle is partially submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.



9. Find the centroid of the region enclosed by $y = 4x - x^2$ and $y = 0$.

10. Find the centroid of the region enclosed by $y = \sin x$, $x = 0$, $x = \pi$, and $y = 0$.

11. Find the centroid of the region enclosed by $x = \sqrt{4 - y^2}$ and $x = 0$.