Test #2 Review Exercise Answers

1.
   a. $\ln(x^2+1) - \frac{1}{2(x^2+1)} - \frac{3}{2}\tan^{-1}x - \frac{3x}{2(x^2+1)} + C$ (the partial fraction decomposition looks like this: $\frac{2x}{x^2+1} + \frac{x-3}{(x^2+1)^2}$)
   b. $-3\ln|x-1| + \ln|x+3| + \ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$ (the partial fraction decomposition looks like this: $-\frac{3}{x-1} + \frac{1}{x+3} + \frac{2x+1}{x^2+4}$)
   c. $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{3}}\right) + C$
   d. $\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2} - 4\sqrt{\sqrt{x}+1} + C$
   e. $x - \frac{1}{2}\ln(1+e^{2x}) + C$
   f. $\frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C$
   g. $\frac{1}{4}$ (that is, the integral converges to $\frac{1}{4}$)
   h. $\frac{\pi}{3}$ (≈ 1.0472)
   i. diverges
   j. 0 (setup: $\lim_{t\to1^-}\int_0^t \frac{1}{(x-1)^{1/3}}\,dx + \lim_{t\to1^+}\int_t^2 \frac{1}{(x-1)^{1/3}}\,dx$)
   k. $\frac{14\sqrt{2}}{3}$ (setup: $\lim_{t\to3^-}\int_1^t \frac{x}{\sqrt{3-x}}\,dx$)
2.
   a. $\int_1^3 x\ln x\,dx \approx 2.966569$, $|E_T| \le \frac{1}{24} = 0.041\overline{6}$
   b. $n = 259$
   c. $\int_1^3 x\ln x\,dx \approx 2.944021$, $|E_S| \le \frac{1}{720} = 0.0013\overline{8}$
   d. $n = 14$
   e. $\int_1^3 x\ln x\,dx \approx 2.943755$
3.
   a. Since $\frac{x^2-x-3}{\sqrt{x^8+x^5+2}} < \frac{x^2}{\sqrt{x^8}} = \frac{x^2}{x^4} = \frac{1}{x^2}$, and $\int_1^\infty \frac{1}{x^2}\,dx$ converges (since $p=2$), it follows by the Comparison Test that $\int_1^\infty \frac{x^2-x-3}{\sqrt{x^8+x^5+2}}\,dx$ converges.
   b. Since $\frac{x^2+2}{(x-1)^3} > \frac{x^2}{x^3} = \frac{1}{x}$, and $\int_2^\infty \frac{1}{x}\,dx$ diverges (since $p=1$), it follows by the Comparison Test that $\int_2^\infty \frac{x^2+2}{(x-1)^3}\,dx$ diverges.
   c. Since $\frac{\sin^2 x+x+x^5}{\sqrt{x^{12}-5x^7-23x}} > \frac{x^5}{\sqrt{x^{12}}} = \frac{x^5}{x^6} = \frac{1}{x}$, and $\int_3^\infty \frac{1}{x}\,dx$ diverges (since $p=1$), it follows by the Comparison Test that $\int_3^\infty \frac{\sin^2 x+x+x^5}{\sqrt{x^{12}-5x^7-23x}}\,dx$ diverges.
   d. Since $\frac{\sqrt{x}-1}{x^3+3x^2} < \frac{\sqrt{x}}{x^3} = \frac{1}{x^{5/2}}$, and $\int_2^\infty \frac{1}{x^{5/2}}\,dx$ converges (since $p=\frac{5}{2}$), it follows by the Comparison Test that $\int_2^\infty \frac{\sqrt{x}-1}{x^3+3x^2}\,dx$ converges.
4.
   a. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{2}+1}\right) \approx 0.435584$
   b. 45

5. $\frac{\pi\cdot 10^{3/2}-\pi}{27} \approx 3.5631$

6. $\frac{208\pi}{9}$

7. 4237 lb (here's a possible setup: $\int_{-4}^{0} 62.5(1-y)\cdot 2\sqrt{16-y^2}\,dy$)

8. 243911 N (here's a possible setup: $\int_{0}^{4} 1000\cdot 9.8\cdot(4-y)\left(\frac{12-2y}{3}\right)dy$)

9. $\left(2,\frac{8}{5}\right)$ (here are the setups: $\bar{x}=\frac{1}{A}\int_0^4 x(4x-x^2)\,dx=2$; $\bar{y}=\frac{1}{A}\int_0^4\left(\frac{4x-x^2}{2}\right)(4x-x^2)\,dx=\frac{8}{5}$; $A=\int_0^4(4x-x^2)\,dx=\frac{32}{3}$; also, note that you can get $\bar{x}$ just from the symmetry of the region)

10. $\left(\frac{\pi}{2},\frac{\pi}{8}\right)$ (here are the setups: $\bar{x}=\frac{1}{A}\int_0^{\pi} x\sin x\,dx=\frac{\pi}{2}$; $\bar{y}=\frac{1}{A}\int_0^{\pi}\left(\frac{\sin x}{2}\right)(\sin x)\,dx=\frac{\pi}{8}$; $A=\int_0^{\pi}\sin x\,dx=2$; also, note that you can get $\bar{x}$ just from the symmetry of the region)

11. $\left(\frac{8}{3\pi},0\right)$ (here are the setups: $\bar{x}=\frac{1}{A}\int_{-2}^{2}\left(\frac{\sqrt{4-y^2}}{2}\right)\left(\sqrt{4-y^2}\right)dy$; $\bar{y}=\frac{1}{A}\int_{-2}^{2} y\sqrt{4-y^2}\,dy=0$; $A=\int_{-2}^{2}\sqrt{4-y^2}\,dy=2\pi$; note that you can get $\bar{y}$ by symmetry, and you can get A by the area of half a circle: $\frac{1}{2}\pi(2)^2$)