Math 181 - Test #2 Info and Review Exercises

Fall 2017, Prof. Beydler

Test Info

- Date: Wednesday, October 25, 2017
- Will cover sections 7.4, 7.5, 7.7, 7.8, 8.1-8.3.
- You'll have the entire class to finish the test.
- For this test, you'll need a **scientific calculator**.
- No notes, no books, no phones, no smart watches during the test.
- There will be a seating chart for the test.
- Where to get help as you're studying:
  - Office hours
  - TMARC, LAC, or other tutoring centers
  - E-mail me at dbeydler@mtsac.edu

Here are **some** of the formulas/concepts that you'll need to know:

Partial fraction decomposition

Linear factor $(x-r)$: $\frac{A}{x-r}$

Repeated linear factor $(x-r)^m$: $\frac{A_1}{x-r}+\frac{A_2}{(x-r)^2}+\cdots+\frac{A_m}{(x-r)^m}$

Irreducible quadratic factor x^2+px+q: $\frac{Ax+B}{x^2+px+q}$

Repeated irreducible quadratic factor $(x^2+px+q)^n$: $\frac{B_1x+C_1}{(x^2+px+q)}+\frac{B_2x+C_2}{(x^2+px+q)^2}+\cdots+\frac{B_nx+C_n}{(x^2+px+q)^n}$

Improper integrals (Type I)

$$\int_a^\infty f(x)\,dx=\lim_{t\to\infty}\int_a^t f(x)\,dx$$

$$\int_{-\infty}^b f(x)\,dx=\lim_{t\to-\infty}\int_t^b f(x)\,dx$$

$$\int_{-\infty}^\infty f(x)\,dx=\int_{-\infty}^c f(x)\,dx+\int_c^\infty f(x)\,dx$$

Improper integrals (Type II)

$$\int_a^b f(x)\,dx=\lim_{t\to a^+}\int_t^b f(x)\,dx \qquad \text{(discontinuity at } x=a)$$

$$\int_a^b f(x)\,dx=\lim_{t\to b^-}\int_a^t f(x)\,dx \qquad \text{(discontinuity at } x=b)$$

$$\int_a^b f(x)\,dx=\int_a^c f(x)\,dx+\int_c^b f(x)\,dx \qquad \text{(discontinuity at } x=c)$$

$\frac{1}{x^p}$ integrals

$\int_1^{\infty} \frac{1}{x^p}\, dx$ converges if $p > 1$ and diverges if $p \leq 1$.

Comparison Test

Suppose f and g are continuous on $[a, \infty)$ and $0 \leq f(x) \leq g(x)$ in $[a, \infty)$.

If $\displaystyle\int_a^{\infty} g(x)\, dx$ converges, then $\displaystyle\int_a^{\infty} f(x)\, dx$ converges.

If $\displaystyle\int_a^{\infty} f(x)\, dx$ diverges, then $\displaystyle\int_a^{\infty} g(x)\, dx$ diverges.

Arc length

$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (or $L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$)

Surface area

$S = \int 2\pi r\, ds$ (often $S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ or $S = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$)

Hydrostatic force

$P_i = \rho g d,\ F_i = P_i A_i$

Centroid (center of mass with uniform density)

$\overline{x} = \frac{1}{A} \int \tilde{x}\, dA$ $\overline{y} = \frac{1}{A} \int \tilde{y}\, dA$ $(\tilde{x}, \tilde{y})$is the centroid of a thin strip, and $dA = (\text{length of strip}) \cdot dx$

I'll give you these formulas if you need them:

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + C$$

Trapezoidal Rule

$$\int_a^b f(x)\, dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

$|E_T| \leq \frac{M(b-a)^3}{12n^2}$ (M is any upper bound of $|f''|$ on $[a, b]$, and n is # of subintervals)

Simpson's Rule

$$\int_a^b f(x)\, dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$|E_S| \leq \frac{M(b-a)^5}{180n^4}$ (M is any upper bound of $\left|f^{(4)}\right|$ on $[a, b]$, and n is # of subintervals)

Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and hand them in at the test, you can earn up to 3% extra credit towards your test (depending on neatness and completeness)! It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the test. Please review the notes, practice problems, previous quizzes, and homework problems to fully prepare for the test.

1. Find the following integrals. If the integral diverges, write "diverges."
   a. $\int \frac{2x^3+3x-3}{x^4+2x^2+1} dx$
   b. $\int \frac{-5x^2-12x-43}{(x-1)(x+3)(x^2+4)} dx$
   c. $\int \frac{\sqrt{x}}{x^2+3x} dx$
   d. $\int \frac{1}{\sqrt{\sqrt{x}+1}} dx$
   e. $\int \frac{1}{1+e^{2x}} dx$
   f. $\int x^3 e^{x^2} dx$
   g. $\int_0^\infty xe^{-2x} dx$
   h. $\int_{-\infty}^\infty \frac{dx}{x^2+9}$
   i. $\int_{-\infty}^0 \frac{xdx}{\sqrt{4-x}}$
   j. $\int_0^2 \frac{dx}{(x-1)^{1/3}}$
   k. $\int_1^3 \frac{xdx}{\sqrt{3-x}}$

2. Consider the integral $\int_1^3 x \ln x \, dx$.
   a. Use the Trapezoidal Rule with $n = 4$ steps to approximate the integral, and then estimate the error in the approximation.
   b. How large does n need to be to guarantee that the approximation from part (a) is accurate to within 0.00001?
   c. Use Simpson's Rule with $n = 4$ steps to approximate the integral, and then estimate the error in the approximation.
   d. How large does n need to be to guarantee that the approximation from part (c) is accurate to within 0.00001?
   e. Evaluate the integral directly.

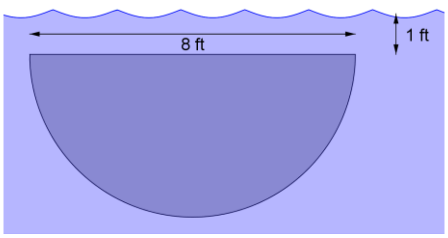
3. Show that each of the following integrals either converge or diverge using the Comparison Test.
   a. $\int_1^\infty \frac{x^2-x-3}{\sqrt{x^8+x^5+2}} dx$
   b. $\int_2^\infty \frac{x^2+2}{(x-1)^3} dx$
   c. $\int_3^\infty \frac{\sin^2 x+x+x^5}{\sqrt{x^{12}-5x^7-23x}} dx$
   d. $\int_2^\infty \frac{\sqrt{x}-1}{x^3+3x^2} dx$

4. Find the length of each of the following curves.
   a. $y = \ln(\cos x)$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$
   b. $y = \frac{2}{3}(x^2+1)^{3/2}$ from $x = 1$ to $x = 4$

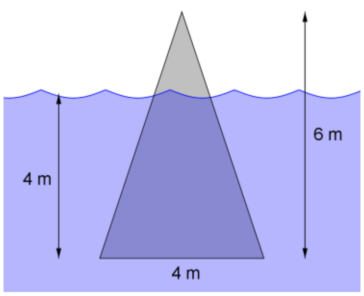
5. Find the area of the surface generated by revolving the curve $x = y^3$ from $y = 0$ to $y = 1$ about the y-axis.

6. Find the area of the surface generated by revolving the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = 1$ to $x = 3$ about the x-axis.

7. A semicircular vertical plate is submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.



8. A vertical plate in the shape of an isosceles triangle is partially submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.



9. Find the centroid of the region enclosed by $y = 4x - x^2$ and $y = 0$.

10. Find the centroid of the region enclosed by $y = \sin x$, $x = 0$, $x = \pi$, and $y = 0$.

11. Find the centroid of the region enclosed by $x = \sqrt{4 - y^2}$ and $x = 0$.