

Test #1 Review Exercise Answers

- $1 - \frac{1}{e}$ (or $\frac{e-1}{e}$) (Set-up: $\int_1^2 e^{y-2} dy$)
- $2 \ln(\sqrt{2} - 1) + 3\sqrt{2} - \ln(2 - \sqrt{3}) - \sqrt{3} - \ln(\sqrt{2} + 1)$ (≈ 1.183);
Set-up: $\int_{\pi/6}^{\pi/4} (\csc x - 2 \sin x) dx + \int_{\pi/4}^{3\pi/4} (2 \sin x - \csc x) dx$
- $\frac{16}{5\sqrt{3}}$ (Set-up: $\int_0^2 \sqrt{3}(2x - x^2)^2 dx$)
- $\frac{4\pi}{3}$ (Set-up: $\int_{-2}^2 \frac{1}{2}\pi \left(\frac{\sqrt{4-y^2}}{2}\right)^2 dy$)
- $\frac{81\pi}{4}$
Washer method set-up: $\pi \int_1^4 \left(x^2 - \left(\frac{1}{x}\right)^2\right) dx$
Shell method set-up: $\int_{1/4}^1 2\pi y \left(4 - \frac{1}{y}\right) dy + \int_1^4 2\pi y(4 - y) dy$
- 192π
Shell method set-up: $\int_0^8 2\pi x \cdot x^{2/3} dx$
Washer method set-up: $\int_0^4 \left(\pi \cdot 8^2 - \pi(y^{3/2})^2\right) dy$
- $\frac{13\pi}{2}$ (Shell method set-up: $\int_1^2 2\pi(3-x)x^2 dx$)
- $\frac{\pi}{6}$ (Shell method set-up: $\int_0^1 2\pi(1-y)(y-y^2) dy$)
- 67.5 ft-lb ($k = 15 \frac{\text{lb}}{\text{ft}}$)
- $(k = 2 \frac{\text{N}}{\text{m}})$
 - 9 J
 - 20 m
- 1470 J (Work to pull up chain: $\int_0^{10} 2 \cdot 9.8 \cdot y dy = 980 \text{ J}$; Work to pull up 5-kg weight: $5 \cdot 9.8 \cdot 10 = 490 \text{ J}$)
- 400 ft-lb (Work to pull up rope: $\int_0^{20} 0.5 \cdot y dy = 100 \text{ ft-lb}$; Work to pull up bucket: $15 \cdot 20 = 300 \text{ ft-lb}$)
- 69688.9 J (Set-up: $\int_0^4 1000 \cdot \left(\frac{y}{3}\right)^2 \cdot 9.8 \cdot (6-y) dy$)
- 2369920 J (Set-up: $\int_{-2}^0 1000 \cdot 10 \cdot 2\sqrt{4-y^2} \cdot 9.8 \cdot (3-y) dy$)
- $\frac{8}{\sqrt{3}\pi}$
- $\frac{e^2+1}{4(e-1)}$

17.

a. $x \tan^{-1} x - \frac{1}{2} \ln(x^2 + 1) + C$ (integration by parts)

b. $\frac{16}{3} \ln 4 - \frac{28}{9} \approx 4.2825$ (integration by parts)

c. $x^2 \sin x + 2x \cos x - 2 \sin x + C$ (integration by parts)

d. $-\frac{\cos^5 x}{5} + \frac{2 \cos^7 x}{7} - \frac{\cos^9 x}{9} + C$ (trig integral)

e. $\frac{\sec^5 x}{5} - \frac{2 \sec^3 x}{3} + \sec x + C$ (trig integral)

f. $\frac{1}{8} x - \frac{1}{32} \sin 4x + C$ (trig integral)

g. $\ln \left| \frac{x}{4} + \frac{\sqrt{x^2 - 16}}{4} \right| + C$ (let $x = 4 \sec \theta$; the integral should simplify to $\int \sec \theta d\theta$)

h. $\frac{x}{\sqrt{9-x^2}} - \sin^{-1} \left(\frac{x}{3} \right) + C$ (let $x = 3 \sin \theta$; the integral should simplify to $\int \frac{\sin^2 \theta}{\cos^2 \theta} d\theta$)

i. $\ln \left| \frac{\sqrt{x^2 + 4x + 8}}{2} + \frac{x+2}{2} \right| + C$ (complete square $(x+2)^2 + 4$; let $x+2 = 2 \tan \theta$; the integral should simplify to $\int \sec \theta d\theta$)