Series

Geometric series: $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^n$
If $|r| < 1$, series converges to $\frac{a}{1-r}$.
If $|r| \geq 1$, series diverges.

***p*-series:** $\sum \frac{1}{n^p}$
If $p > 1$, series converges.
If $p \leq 1$, series diverges.

Test for Divergence
If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
(Note: If $\lim_{n\to\infty} a_n = 0$, then you don't know anything. Try something else.)

The Integral Test (need $a_n = f(n)$, and $f(x)$ continuous, positive, and decreasing for all $x \geq N$)
$\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)\,dx$ both converge or both diverge.

The Comparison Test (need $a_n \geq 0$ and $b_n \geq 0$)
If $a_n \leq b_n$ for all $n > N$ and if $\sum b_n$ converges, then the smaller $\sum a_n$ also converges.
If $b_n \leq a_n$ for all $n > N$ and if $\sum b_n$ diverges, then the bigger $\sum a_n$ also diverges.

The Limit Comparison Test (need $a_n > 0$ and $b_n > 0$ for all $n \geq N$)
1. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then $\sum a_n$ and $\sum b_n$ both converge or both diverge.
2. If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.
3. If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

The Alternating Series Test (AST)
$\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ converges if:
1. $b_n > 0$ for all $n \geq N$
2. $b_{n+1} \leq b_n$ for all $n \geq N$
3. $b_n \to 0$

The Absolute Convergence Test
If $\sum |a_n|$ converges, then $\sum a_n$ converges.

The Ratio Test
Suppose $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$.
If $L < 1$, then $\sum a_n$ converges absolutely.
If $L > 1$ (or L infinite), then $\sum a_n$ diverges.
If $L = 1$, then test inconclusive (try something else).

The Root Test
Suppose $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$.
If $L < 1$, then $\sum a_n$ converges absolutely.
If $L > 1$ (or L infinite), then $\sum a_n$ diverges.
If $L = 1$, then test inconclusive (try something else).