Computer Lab #1

(due December 6, 2017)

For the computer labs, you'll be exploring how to use a free, open source math program called GeoGebra. GeoGebra is a Computer Algebra System (CAS), which means it can manipulate math expressions in symbolic form (with variables). Other popular CAS's include Mathematica, Maple, and Maxima. (MMM! 😉)

Software you'll need

1. **GeoGebra:** Visit www.geogebra.org and click on "GeoGebra Classic." GeoGebra should open up in your browser. You can also download a desktop version by clicking on "Downloads" and then scrolling down to "GeoGebra Classic 6." Either one you use will be fine, though the instructions in this document will look more like the online version. However, the desktop version is more reliable and powerful.
2. **Document Editor:** For this lab, you'll also need a document editor, such as Microsoft Word, Google Docs, etc. Just pick your favorite one.

How to insert GeoGebra images into your report

- Use screen capture software or plugins (for example, Snipping Tools on Windows, Nimbus Screenshot for Chrome, etc.), and then paste the image into your favorite document editor.
- GeoGebra Classic online version: Download the image by going to the hamburger menu (☰) on the top right, then "File," then "Download as...," then "png." Then drag the PNG file into your favorite document editor.
- GeoGebra Classic 6 desktop version: Go to "Edit," then "Graphics View to Clipboard," then paste the image into your favorite document editor (usually Ctrl-V, or Edit->Paste).

Please visit the class website to view a sample lab page to see what a lab report should look like. Remember that you'll need to include any GeoGebra used to answer the problems. The commands can be copied from the input bar on the left side and pasted into your report.

Important things to know for this assignment

- Here's how you can draw a slope field:

  `f(x,y)=cos(x/4)+y/4` ← This defines $f(x,y)$ to be $\cos\left(\frac{x}{4}\right)+\frac{y}{4}$

  `SlopeField(f,30)` ← This shows a slope field for $\frac{dy}{dx}=f(x,y)$

  `SolveODE(f, (1,0))` ← This shows the particular solution for $\frac{dy}{dx}=f(x,y)$ and $y(1)=0$

- Here are a couple of ways you can adjust the x scale and y scale:
  - Click on and then . Under the "Basic" tab, you can adjust the minimum and maximum x and y values that you see.

- Hold shift and drag x-axis or y-axis to adjust the scale.
- Resize your browser or desktop application window.
- If you need help, here's a short tutorial about GeoGebra: https://www.geogebra.org/b/P9fSOxh1

In this lab, you're going to dive into differential equations (DE's).

Slope Fields

1. Consider the DE: $y' = (\sin x)(\sin y)$.
   a. Using GeoGebra, plot a slope field with $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$.
   b. By hand, find the particular solution that satisfies the initial value problem: $y' = (\sin x)(\sin y)$, $y(0) = \frac{\pi}{4}$. Be sure you solve explicitly for y. (Hint: $\csc x + \cot x = \frac{1}{\tan\left(\frac{x}{2}\right)}$)
   c. Over the slope field you plotted in part (a), use GeoGebra to plot the solution curve that you found in part (b) with $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$. Does the solution flow along with the slope field?

Autonomous DEs

An **autonomous** differential equation is a DE that can be written in the form $y' = f(y)$. That is, y' only depends on y, not x. So, when you plot the slope field (part (a) below), notice that if you look across the slope field horizontally (constant y), the slopes will be the same.

2. Use the autonomous DE $\frac{dy}{dx} = 0.01y(1-y)\left(y - \frac{1}{2}\right)$ to answer the questions below.
   a. Using GeoGebra, plot a slope field with $0 \leq x \leq 300$ and $-2 \leq y \leq 2$, and plot solution curves with each of the following initial conditions (using the `SolveODE` command): $y(0) = -1$, $y(0) = \frac{1}{4}$, $y(0) = \frac{3}{4}$, $y(0) = 2$
   b. Suppose the DE models the population of people on an island. So, y represents the population (in millions of people), x represents time (in years). Based on the model, if the population is 2 million people when $x = 0$, what will the population be 40 years later? Use GeoGebra to answer this question. The output of `SolveODE` will be a function, so you can just plug 40 into that function.
   c. Based on the model as explained in part (b), what would happen if the population fell below 500,000 people? How can you tell by looking at the DE?