Strategy for Integration

Given an integral, which technique should you use? Here are some things to try:

1. Simplify first
2. Substitution
    - If $u = g(x)$ appears, as well as $du = g'(x)dx$, then try substitution.
3. Is the integrand one of these?
    - Trig functions: 7.2 techniques (trig integrals)
    - Rational functions: 7.4 techniques (partial fractions)
    - A product of functions: try integration by parts
    - Radicals ($\sqrt{x^2 + a^2}$, $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$): 7.3 techniques (trig substitution)
4. If nothing works so far…
    - Try substitution again
    - Try parts again
    - Manipulate integrand (rationalize denominator, use trig identities, etc.)

Ex 1.

Evaluate the following integral.

$\int \frac{dx}{1-\cos x}$

Ex 2.

Evaluate the following integral.

$\int \frac{1-e^x}{1+e^x}\,dx$

Ex 3.

Evaluate the following integral.

$\int e^{\sqrt{x}}\,dx$

Ex 4.

Evaluate the following integral.

$\int \frac{1}{\sqrt{\sqrt{x}+1}}dx$

Practice

1. Evaluate each integral.

   a) $\int \frac{\cos x}{\sin^2 x - \sin x}dx$

b) $\int \frac{\tan^3 x}{\cos^3 x}\, dx$

c) $\int \left(\sqrt{x} + 2\right)^{10} dx$

Q: In a zoo with only birds and mammals, there are 30 heads and 80 legs. How many birds and how many mammals are at this zoo?