7.4 – Practice

1. Evaluate each integral.

a) $\int \frac{x^2+8}{x^2-5x+6}\,dx$

$$= \int \left(1 + \frac{5x+2}{x^2-5x+6}\right) dx$$

$$\begin{array}{r} 1 \\ x^2-5x+6 \,\big)\, x^2+0x+8 \\ -(x^2-5x+6) \\ \hline 5x+2 \end{array}$$

$$= \int \left(1 - \frac{12}{x-2} + \frac{17}{x-3}\right) dx$$

$$\frac{5x+2}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$= \boxed{x - 12\ln|x-2| + 17\ln|x-3| + C}$$

$5x+2 = A(x-3) + B(x-2)$

$\underline{x=2:}$ $12 = -A \rightarrow A = -12$

$\underline{x=3:}$ $17 = B$

b) $\int \frac{1}{x(x^2+1)^2}\,dx$ (Hint: For part b, it's probably easier to expand and match coefficients rather than plug in x-values. This often helps with irreducible quadratic factors.)

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$1 = A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x$$

$$1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x) + Dx^2+Ex$$

$$1 = Ax^4+2Ax^2+A+Bx^4+Bx^2+Cx^3+Cx+Dx^2+Ex$$

$$1 = (A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A$$

Match coefficients of like terms:

$$\begin{aligned} A+B&=0 \\ C&=0 \\ 2A+B+D&=0 \\ C+E&=0 \\ A&=1 \end{aligned} \quad \xrightarrow[\text{system}]{\text{Solve}} \quad \begin{aligned} A&=1 \\ B&=-1 \\ C&=0 \\ D&=-1 \\ E&=0 \end{aligned}$$

So, $$\int \frac{1}{x(x^2+1)^2}dx = \int\left[\frac{1}{x} + \frac{-x}{x^2+1} + \frac{-x}{(x^2+1)^2}\right]dx$$

$$= \int \frac{1}{x}dx - \int \frac{x}{x^2+1}dx - \int \frac{x}{(x^2+1)^2}dx$$

$u = x^2+1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$= \ln|x| - \frac{1}{2}\int \frac{1}{u}du - \frac{1}{2}\int \frac{1}{u^2}du$$

$$= \ln|x| - \frac{1}{2}\ln|u| - \frac{1}{2}\left(-\frac{1}{u}\right) + C$$

$$= \boxed{\ln|x| - \frac{1}{2}\ln|x^2+1| + \frac{1}{2(x^2+1)} + C}$$

or, $\ln\frac{|x|}{\sqrt{x^2+1}} + \frac{1}{2(x^2+1)} + C$

Q: What do you get when you expand $(x-a)(x-b)(x-c)\ldots(x-y)(x-z)$?