Trigonometric Integrals

Some trigonometric integrals we know how to evaluate:

$$\int \sec^2 x\, dx = \tan x + C$$

For evaluating other trig integrals, we might need to use trig identities first.

Know these trig formulas!

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Ex 1.

$$\int \sin^3 x \cos^2 x\, dx$$

Ex 2.

$$\int \sin^4 x\, dx$$

For integrals of the form $\int \sin^m x \cos^n x\, dx$ …

…if m or n is **odd**, save one of the odd factors with dx, and then use $\sin^2 x + \cos^2 x = 1$.

…if m and n are both **even**, use $\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$ to lower the powers.

Ex 3.

$$\int \sec^4 x \tan^3 x \, dx$$

Ex 4.

$$\int \tan^3 x \, dx$$

For integrals of the form $\int \mathbf{tan^m}\, \boldsymbol{x}\, \mathbf{sec^n}\, \boldsymbol{x}\, \boldsymbol{dx}$ …

…if n **even**, save a $\sec^2 x$ with the dx, use $\tan^2 x + 1 = \sec^2 x$, then let $u = \tan x$.

…if m is **odd**, save a $\sec x \tan x$, use $\tan^2 x + 1 = \sec^2 x$, then let $u = \sec x$.

…if m is even, use $\tan^2 x + 1 = \sec^2 x$.

…if n is odd, try integration by parts.

(Note: The process is similar for $\int \cot^m x \csc^n x\, dx$.)

Ex 5.

$$\int \sec^3 x\, dx$$

Eliminating Square Roots

Ex 6.

$$\int_0^{\pi/4} \sqrt{1+\cos 4x}\, dx$$

Practice

1. Evaluate each integral.

a) $\int \cos^5 x\, dx$

b) $\int \sin^2 x \, dx$

c) $\int \tan^4 x \, dx$

d) $\int \sqrt{\frac{1-\cos x}{2}} \, dx$ (Assume $0 \leq x \leq 2\pi$.)

Q: The more you take, the more you leave behind. What are they?