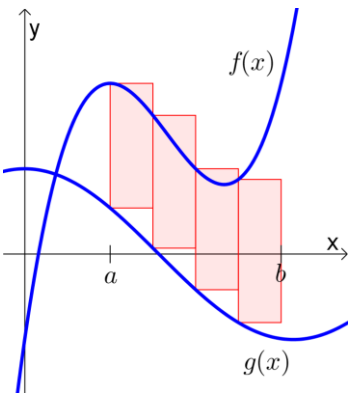
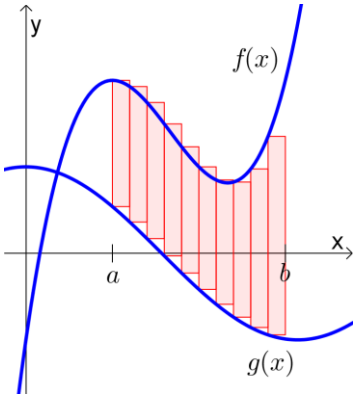
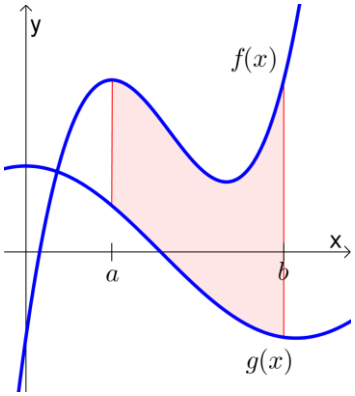
Area Between Curves

Q: How can we find the area between two curves?







$$\sum_{i=1}^{n}[f(x_i)-g(x_i)]\Delta x \xrightarrow[n\to\infty]{} \int_a^b [f(x)-g(x)]\,dx$$

If f and g are continuous with $f(x) \geq g(x)$ on $[a,b]$, then

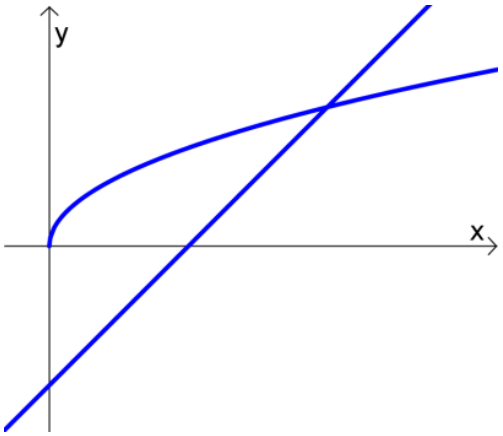
$$\text{Area between curves } = \int_a^b [f(x)-g(x)]\,dx$$

Ex 1.

Find the area of the region enclosed by the curves $y = 2 - x^2$ and $y = -x$.

Ex 2.

Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

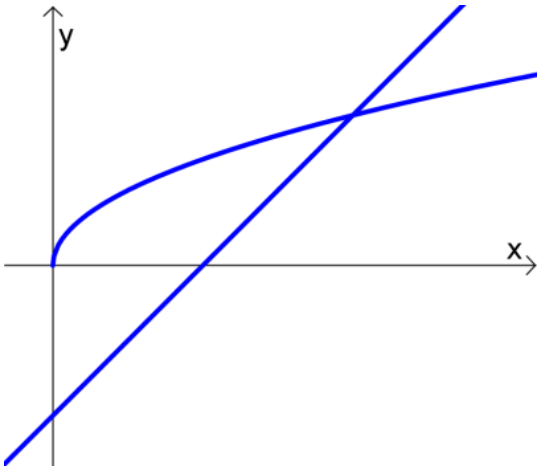


In the last example, it would have been more convenient to integrate with respect to y. The area formula becomes:

$$\text{Area between curves } = \int_c^d [f(y) - g(y)]\, dy$$

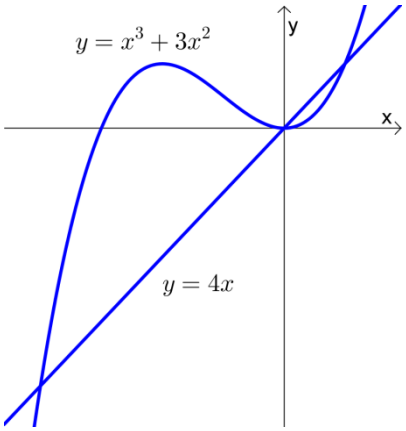
Ex 3.

Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$. But this time, integrate with respect to y.



Practice

1. Find the area of the region(s) enclosed by the line $y = 4x$ and the curve $y = x^3 + 3x^2$.



2. Find the area of the region enclosed by the curves $x - y^2 = 0$ and $x + 2y^2 = 3$. (Hint: Integrate in y direction.)