Representations of Functions as Power Series

Recall that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$ (it's a geometric series, $a = 1, r = x$). Noticing this allows us to represent other functions as power series, too.

Ex 1.

Express $\frac{1}{1+x^2}$ as a power series and find its interval of convergence.

Ex 2.

Express $\frac{x^3}{x+2}$ as a power series and find its interval of convergence.

Note:

You can differentiate a series with $R > 0$ term-by-term, and the result will have the same radius and interval of convergence (except perhaps for the endpoints of the interval of convergence).

Ex 3.

Find a power series representation for $\frac{1}{(1-x)^2}$ and find its radius of convergence.

Note:

You can also integrate a series with $R > 0$ term-by-term, and the result will have the same radius and interval of convergence (except perhaps for the endpoints of the interval of convergence).

Ex 4.

Find a power series representation for $\ln(1 - x)$ and find its radius of convergence.

Ex 5.

Evaluate $\int \frac{1}{1+x^5}\,dx$ as a power series. Then use the power series to approximate $\int_0^{0.5} \frac{1}{1+x^5}\,dx$ correct to within 0.0000001.

Practice

1. Express $\frac{x}{2x^2+1}$ as a power series and find its interval of convergence.

2. Find a power series representation for $\tan^{-1} x$. Then plug in $x = 1$. (Caution: your brain might explode.)

Q: What has one head, one foot, and four legs?