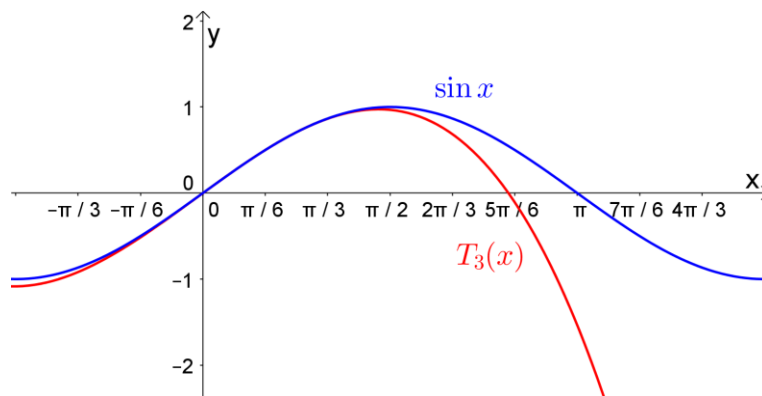


1. Find the Taylor polynomial $T_3(x)$ for $f(x) = \sin x$ centered at $a = \frac{\pi}{6}$. Then use Taylor's Inequality to estimate the accuracy of the $T_3(x)$ when x lies in the interval $0 \leq x \leq \frac{\pi}{3}$.



$$\begin{aligned}
 f(x) &= \sin x & f\left(\frac{\pi}{6}\right) &= \frac{1}{2} \\
 f'(x) &= \cos x & f'\left(\frac{\pi}{6}\right) &= \frac{\sqrt{3}}{2} \\
 f''(x) &= -\sin x & f''\left(\frac{\pi}{6}\right) &= -\frac{1}{2} & \frac{f''\left(\frac{\pi}{6}\right)}{2!} &= -\frac{1}{4} \\
 f'''(x) &= -\cos x & f'''\left(\frac{\pi}{6}\right) &= -\frac{\sqrt{3}}{2} & \frac{f'''\left(\frac{\pi}{6}\right)}{3!} &= -\frac{\sqrt{3}}{12}
 \end{aligned}$$

$$T_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$$

Note: $0 \leq x \leq \frac{\pi}{3} \Rightarrow -\frac{\pi}{6} \leq x - \frac{\pi}{6} \leq \frac{\pi}{6} \Rightarrow \left|x - \frac{\pi}{6}\right| \leq \frac{\pi}{6}$

$$\begin{aligned}
 |R_3(x)| &\leq \frac{M}{4!} \left|x - \frac{\pi}{6}\right|^4 \\
 &\leq \frac{(\sqrt{3}/2)}{24} \left(\frac{\pi}{6}\right)^4 \\
 &\approx \boxed{0.003}
 \end{aligned}$$

Find M:

$$\begin{aligned}
 |f^{(4)}(x)| &= |\sin x| \quad \text{for } 0 \leq x \leq \frac{\pi}{3} \\
 &= \sin x \\
 &\leq \sin \frac{\pi}{3} \\
 &= \frac{\sqrt{3}}{2} \\
 &\quad \leftarrow M
 \end{aligned}$$

Joke: What did the buffalo say to his son when he left for college? Bison!

Bonus Joke: Did you hear the joke about the roof? Never mind, it's over your head!