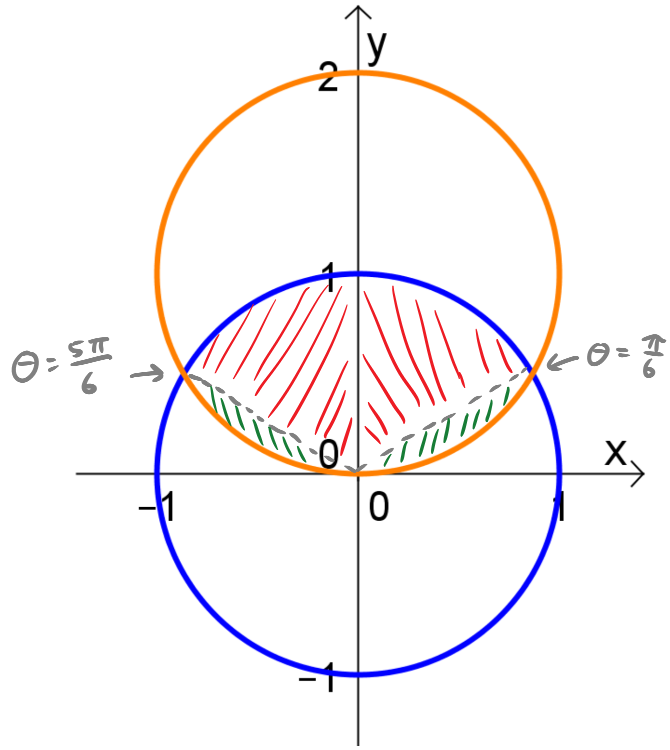
1. Find the area of the region shared by the circles $r = 1$ and $r = 2\sin\theta$.



Intersection points:

$1 = 2\sin\theta$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = 2\int_0^{\frac{\pi}{6}} \frac{1}{2}(2\sin\theta)^2\,d\theta + 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(1)^2\,d\theta$$

$$= 4\int_0^{\frac{\pi}{6}} \sin^2\theta\,d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} d\theta$$

$$= 2\int_0^{\frac{\pi}{6}} (1-\cos 2\theta)\,d\theta + \left[\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

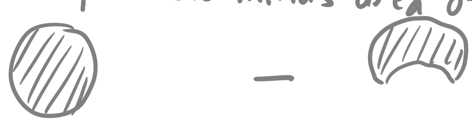
$$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{6}} + \left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$= 2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) + \frac{\pi}{3}$$

$$= \boxed{\frac{2\pi}{3} - \frac{\sqrt{3}}{2}}$$

OR

Think area of top circle minus area of top "crescent".



$$A = \pi(1)^2 - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}\left((2\sin\theta)^2 - (1)^2\right)d\theta$$

$= \ldots$

2. Find the length of the curve $r = \sqrt{1 + \sin 2\theta}$, $0 \le \theta \le \pi\sqrt{2}$.

$$\frac{dr}{d\theta} = \frac{1}{2}(1+\sin 2\theta)^{-1/2} \cdot 2\cos 2\theta = \frac{\cos 2\theta}{\sqrt{1+\sin 2\theta}}$$

$$\text{Length} = \int_0^{\pi\sqrt{2}} \sqrt{\left(\sqrt{1+\sin 2\theta}\right)^2 + \left(\frac{\cos 2\theta}{\sqrt{1+\sin 2\theta}}\right)^2}\, d\theta$$

$$= \int_0^{\pi\sqrt{2}} \sqrt{1+\sin 2\theta + \frac{\cos^2 2\theta}{1+\sin 2\theta}}\, d\theta$$

$$= \int_0^{\pi\sqrt{2}} \sqrt{\frac{1+2\sin 2\theta + \sin^2 2\theta + \cos^2 2\theta}{1+\sin 2\theta}}\, d\theta$$

$$= \int_0^{\pi\sqrt{2}} \sqrt{\frac{2+2\sin 2\theta}{1+\sin 2\theta}}\, d\theta$$

$$= \int_0^{\pi\sqrt{2}} \sqrt{2}\, d\theta$$

$$= \sqrt{2}\left(\pi\sqrt{2}\right)$$

$$= \boxed{2\pi}$$

Q: This is an unusual paragraph. I'm curious how quickly you can find out what is so unusual about it. It looks so plain you would think nothing was wrong with it. In fact, nothing is wrong with it! It is unusual though. Study it, and think about it, but you still may not find anything odd. But if you work at it a bit, you might find out. Try to do so without any coaching!