


Math 180 - Test #3 Info and Review Exercises

Spring 2020, Prof. Beydler

Test Info

- Test #3 Date: Wednesday, May 27, 2020
- Will cover packets #17 through #24.
- You'll have from 9:30am to 2:00pm to finish the test.
- I'll post Test #3 as an assignment on Canvas on the test day at 9:30am.
 - If you have a printer, you can print the test out and write your solutions directly on it.
 - If you have a tablet, please feel free to use it to write directly into the PDF.
 - If you don't have a printer or tablet, feel free to use your own paper. Please put the test problems in order.
 - You can submit your test just like you do for your homework assignments. So, you'll attach the PDF under the Test #3 assignment.
- We're working on the honor system here: no notes, books, phones, or computers during the test.

Formulas and stuff

(Note: Know all of these except for the ones with  next to them, which I'll give you. This list is not meant to include everything you'll need to know on the test.)

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$(cf)' = cf'$$

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = fg' + gf'$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}} \text{ 🌴}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}} \text{ 🌴}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2} \text{ 🌴}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \operatorname{coth} x \text{ 🌴}$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \text{ 🌴}$$

$$\frac{d}{dx}(\operatorname{coth} x) = -\operatorname{csch}^2 x \text{ 🌴}$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \text{ 🌴}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}} \text{ 🌴}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \text{ 🌴}$$

$$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}} \text{ 🌴}$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}} \text{ 🌴}$$

$$\frac{d}{dx}(\operatorname{coth}^{-1} x) = \frac{1}{1-x^2} \text{ 🌴}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

Here are some helpful geometry formulas to know for optimization (packet #18) problems:

Pythagorean Theorem: $a^2 + b^2 = c^2$ (or $(leg)^2 + (leg)^2 = (hypotenuse)^2$)

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Area of rectangle: $A = lw$

Area of circle: $A = \pi r^2$

Area of triangle: $A = \frac{1}{2}bh$

Circumference of circle: $C = 2\pi r = \pi d$

How to get perimeter of any polygon (just add the lengths of the sides).

How to get the surface area of a 3-D surface (just add the areas of the faces/sides).

Volume of a box (also called a rectangular prism): $V = lwh$

Summation Properties

$$\sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$$

$$\sum_{i=1}^n ca_i = c \cdot \sum_{i=1}^n a_i$$

$$\sum_{i=1}^n c = n \cdot c$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Integral Properties

$$\int_b^a f(x) dx = -\int_a^b f(x) dx$$

$$\int_a^a f(x) dx = 0$$

$$\int_a^b kf(x) dx = k \int_a^b f(x) dx$$

$$\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

If $f(x) \geq g(x)$ on $[a, b]$, then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$.

The Fundamental Theorem of Calculus, Part 2: $\int_a^b f(x) dx = F(b) - F(a)$

Substitution: $\int f(g(x))g'(x) dx = \int f(u) du$

Integration by parts: $\int u dv = uv - \int v du$

Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and submit them on Canvas on the test day, you can earn up to 3% extra credit towards your test! It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the test. Please review the notes and homework problems to fully prepare for the test.

1. Find the absolute maximum and minimum values of $f(x) = x - \ln(x^2)$ on the interval $[1,3]$.
2. Find the absolute maximum and minimum values of $f(x) = xe^{-2x}$ on the interval $[-2, 2]$.
3. A rectangle has its two lower corners on the x -axis and its two upper corners on the curve $y = 9 - x^2$.
What are the dimensions of such a rectangle that maximize its area?

4. If you send a package via USPS Express Mail International, the length (the longest dimension) plus the girth (distance around thickest part) can't be more than 79 inches. What's the largest possible volume of a rectangular box with a square base that can be sent by USPS Express Mail International? And what are the dimensions of the box?

5. An open box with a square base is going to be made. The sides of the box will cost \$3 per square foot, and the base will cost \$5 per square foot. What are the dimensions of the box with the largest volume that can be constructed for \$60?

6. Find the most general antiderivative for each of the following functions.

a) $f(x) = 8\sqrt[3]{x} - \frac{2}{x} + x^{9999} - \frac{10}{\sqrt{x}} + \frac{1}{x^3} + x\sqrt{x}$

b) $f(x) = \sin 3x + 2 \sec x \tan x - e^{-x} + 3^x + 19$

c) $f(x) = \frac{2}{1+x^2} - \frac{3}{\sqrt{1-x^2}} - \csc^2 x$

7. A particle is moving with the given data. Find the position of the particle.

$$a(t) = 3 \cos t - 2 \sin t, \quad s(0) = 0, \quad v(0) = 4$$

8. A particle is moving with the given data. Find the position of the particle.

$$a(t) = t^2 - 4t + 6, \quad s(0) = 0, \quad s(1) = 20$$

9. Find f if $f''(x) = 2e^x + 3 \sin x$, $f(0) = 0$, and $f(\pi) = 0$.

10. Estimate the area under the graph of $f(x) = \sqrt{x}$ between $x = 0$ and $x = 4$ using...

a) ...a lower sum with four rectangles of equal width.

b) ...an upper sum with four rectangles of equal width.

c) ...midpoints with four rectangles of equal width.

11. Estimate the area under the graph of $f(x) = e^{-x} + 1$ between $x = -1$ and $x = 1$ using...

a) ...right endpoints with four rectangles of equal width.

b) ...left endpoints with four rectangles of equal width.

c) ...midpoints with four rectangles of equal width.

12. Estimate the distance traveled in 12 hours given the following sample velocities...

Time (hours)	0	2	4	6	8	10	12
Velocity (mph)	60	63	64	50	45	48	70

a) ...using left-endpoint values.

b) ...using right-endpoint values.

13. Evaluate the following integrals using Riemann sums with right endpoints.

a) $\int_0^3 (2x + x^2) dx$

b) $\int_0^4 (x^2 - 4x + 2) dx$

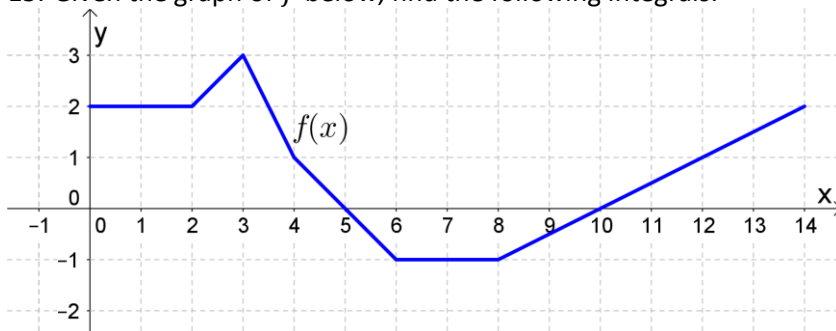
14. Suppose that $\int_1^5 f(x) dx = -2$, $\int_7^1 f(x) dx = 10$, and $\int_1^5 g(x) dx = 3$. Find the following.

a) $\int_5^1 3f(x) dx$

b) $\int_1^5 [g(x) - 4f(x)] dx$

c) $\int_5^7 f(x) dx$

15. Given the graph of f below, find the following integrals.



a) $\int_0^1 f(x) dx$

b) $\int_0^3 f(x) dx$

c) $\int_3^6 f(x) dx$

d) $\int_4^6 f(x) dx$

e) $\int_4^{10} f(x) dx$

f) $\int_0^{14} f(x) dx$

16. Graph the following integrands and use the area under the graph to evaluate the integral.

a) $\int_{-1}^6 (|x - 3| + 1) dx$

b) $\int_0^2 (3 + \sqrt{4 - x^2}) dx$

17. Evaluate the following integrals.

a) $\int_4^9 (2x\sqrt{x} - 3) dx$

b) $\int_0^{\pi/8} \sin 2x dx$

c) $\int_{\pi/6}^{\pi/2} \csc^2 x \, dx$

d) $\int_0^1 \frac{5}{1+x^2} \, dx$

e) $\int_0^{\pi/3} (2x - \sec x \tan x) \, dx$

f) $\int_{1/2}^1 \frac{1}{2x} \, dx$

g) $\int_0^1 e^{3x} dx$

h) $\int_0^7 \frac{1}{\sqrt[3]{x+1}} dx$

i) $\int \frac{4x^2+2}{(2x^3+3x)^4} dx$

j) $\int e^{2 \cos x} \cdot \sin x \, dx$

k) $\int 3xe^{x^2+2} \, dx$

l) $\int x^2 e^{-2x} \, dx$

m) $\int x^2 \ln x \, dx$

n) $\int e^{2x} \cos x \, dx$

o) $\int x\sqrt{2x+1} \, dx$

18. Suppose the velocity function of a particle is $v(t) = t^2 - 2t$ (in meters per second). Find the distance traveled by the particle during the time period $-2 \leq t \leq 5$.