Test #3 (Part 2 – Scientific Calculator Okay) Name: ______________________________

Math 180, Prof. Beydler Wednesday, November 28, 2018

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (3 points) Find the absolute maximum and minimum values of $f(x) = xe^{-x}$ on the interval $[0, 3]$.

$f'(x) = x(-e^{-x}) + 1 \cdot e^{-x} = e^{-x}(-x+1)$

Absolute maximum value: $\frac{1}{e}$

Absolute minimum value: 0

$\underline{f' = 0:}$

$e^{-x}(-x+1) = 0$

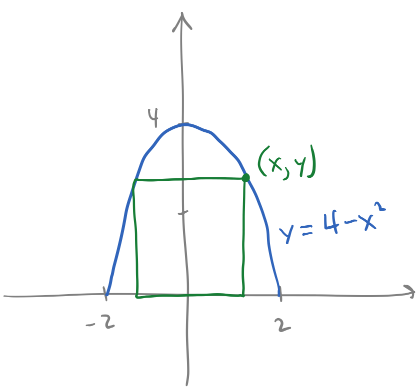
$\downarrow$

$x = 1$

x	$f(x)$
1	$\frac{1}{e} \approx 0.37$
0	0
3	$\frac{3}{e^3} \approx 0.15$

2. (4 points) A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 4 - x^2$. What are the dimensions of such a rectangle that maximize its area?

Dimensions of rectangle: $\frac{4}{\sqrt{3}}$ by $\frac{8}{3}$



$A(x, y) = 2x \cdot y$

$A(x) = 2x(4 - x^2)$

$= 8x - 2x^3$

$A'(x) = 8 - 6x^2$

$\underline{A'(x) = 0:}$

$8 - 6x^2 = 0$

$x^2 = \frac{8}{6} = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

$y = 4 - \left(\frac{2}{\sqrt{3}}\right)^2$

$= 4 - \frac{4}{3}$

$= \frac{8}{3}$

3. (3 points) A particle is moving with the given data. Find the position of the particle.
$a(t) = 3t^2 - 5,\ s(0) = 0,\ s(1) = 4$

$v(t) = t^3 - 5t + C$

$s(t) = \frac{t^4}{4} - \frac{5}{2}t^2 + Ct + D$

$s(0)=0:$ $D=0$

$s(1)=4:$ $\frac{1}{4} - \frac{5}{2} + C + 0 = 4$

$C = \frac{25}{4}$

Position $s(t) =$ $\frac{t^4}{4} - \frac{5}{2}t^2 + \frac{25}{4}t$

4. (2 points) Estimate the distance traveled in 24 seconds given the following sample velocities using right-endpoint values. Be sure to write the units for your answer.

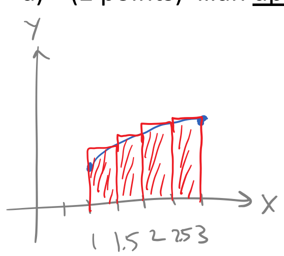
Time (s)	0	6	12	18	24
Velocity (m/s)	5	4.3	4.6	5.1	2.5

$6(4.3 + 4.6 + 5.1 + 2.5)$

Answer: 99 m

5. Estimate the area under the graph of $f(x) = \sqrt{x+1}$ between $x = 1$ and $x = 3$ using…

a) (2 points) …an upper sum with four rectangles of equal width. Write your answer to 6 decimal places.

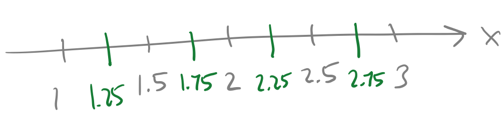


$\Delta x = \frac{3-1}{4} = \frac{1}{2}$

$\frac{1}{2}\left(\sqrt{1.5+1} + \sqrt{2+1} + \sqrt{2.5+1} + \sqrt{3+1}\right)$

Answer: 3.592009

b) (2 points) …midpoints with four rectangles of equal width. Write your answer to 6 decimal places.



$\frac{1}{2}\left(\sqrt{1.25+1} + \sqrt{1.75+1} + \sqrt{2.25+1} + \sqrt{2.75+1}\right)$

Answer: 3.448790

6. Suppose that $\int_3^{-2} f(x)\,dx = -2$ and $\int_3^5 f(x)\,dx = 4$. Find the following.

a) (1 point) $\int_{-2}^{3} f(x)\,dx$

Answer: 2

b) (1 point) $\int_{-2}^{5} f(x)\,dx$

$$= \int_{-2}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$$

$$= 2 + 4$$

Answer: 6

7. (5 points) Evaluate the following integral <u>using Riemann sums with right endpoints</u>.

$$\int_0^2 (x^3 + 2x)\,dx \qquad \Delta x = \frac{2-0}{n} = \frac{2}{n}, \qquad x_i = a + i\Delta x = 0 + i\left(\frac{2}{n}\right) = \frac{2i}{n}$$

Answer: 8

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^3 + 2\left(\frac{2i}{n}\right)\right]\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{16i^3}{n^4} + \frac{8i}{n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{16}{n^4}\sum_{i=1}^{n} i^3 + \frac{8}{n^2}\sum_{i=1}^{n} i\right)$$

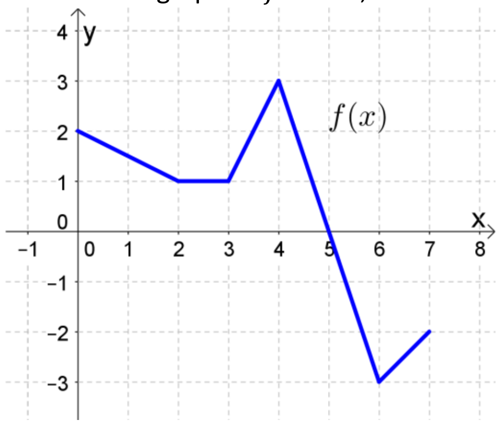
$$= \lim_{n\to\infty} \left(\frac{16}{n^4}\cdot\left[\frac{n(n+1)}{2}\right]^2 + \frac{8}{n^2}\cdot\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{4(n^2+2n+1)}{n^2} + \frac{4(n+1)}{n}\right)$$

$$= \lim_{n\to\infty} \left(4 + \frac{8}{n} + \frac{4}{n^2} + 4 + \frac{4}{n}\right)$$

$$= 8$$

8. Given the graph of f below, find the following integrals.



a) (1 point) $\int_0^2 f(x)\,dx =$ <u>3</u>

b) (1 point) $\int_2^4 f(x)\,dx =$ <u>3</u>

c) (1 point) $\int_4^7 f(x)\,dx =$ <u>−2.5</u>

9. (4 points) Suppose the velocity function of a particle is $v(t) = t^2 + 3t - 4$ (in meters per second). Find the distance traveled by the particle during the time period $0 \le t \le 3$. Be sure to write units for your answer.

$t^2 + 3t - 4 = 0$

$(t+4)(t-1) = 0$

$t = -4 \qquad t = 1$

0 1 3

$v(t)$ − 0 +

Distance traveled: $\frac{89}{6}$ m

$$\text{Distance} = \int_0^3 |v(t)|\,dt$$

$$= \int_0^1 -(t^2+3t-4)\,dt + \int_1^3 (t^2+3t-4)\,dt$$

$$= \left[-\frac{t^3}{3} - \frac{3t^2}{2} + 4t\right]_0^1 + \left[\frac{t^3}{3} + \frac{3t^2}{2} - 4t\right]_1^3$$

$$= -\frac{1}{3} - \frac{3}{2} + 4 + 9 + \frac{27}{2} - 12 - \left(\frac{1}{3} + \frac{3}{2} - 4\right)$$

$$= -\frac{2}{3} + \frac{21}{2} + 5$$

$$= -\frac{4}{6} + \frac{63}{6} + \frac{30}{6}$$

$$= \frac{89}{6}$$

Note: Be sure to double check your work. And remember to turn in your homework and extra credit! ☺

Here are some formulas that I promised to give:

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$