_____ / 50 total points (both parts)

Test #3 (Part 1 – No Calculator)

Name: ______________________________

Math 180, Prof. Beydler

Wednesday, November 28, 2018

Directions: Show all work. No calculator, books, or notes. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. When you're finished with Part 1, please turn it in, take a bathroom break, get your calculator out, and start Part 2. Good luck!

1. (3 points) Find the most general antiderivative for $f(x) = \frac{3}{\sqrt[3]{x}} - \frac{2}{5x} - \cos 2x - 4\csc^2 x + \frac{2}{\sqrt{1-x^2}}$

Answer: $\frac{9}{2}x^{2/3} - \frac{2}{5}\ln|x| - \frac{1}{2}\sin 2x + 4\cot x + 2\sin^{-1}x + C$

$$f(x) = 3x^{-1/3} - \frac{2}{5}\cdot\frac{1}{x} - \cos 2x - 4\csc^2 x + 2\cdot\frac{1}{\sqrt{1-x^2}}$$

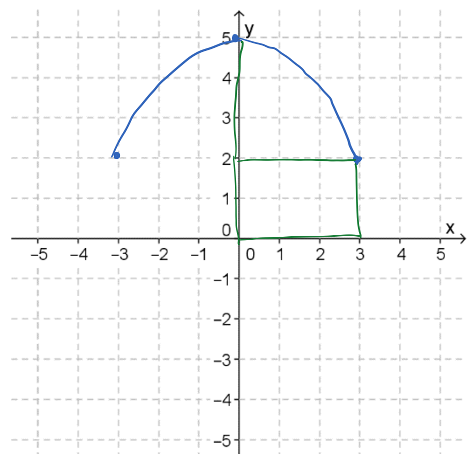
$$F(x) = 3\cdot\frac{3}{2}x^{2/3} - \frac{2}{5}\ln|x| - \frac{1}{2}\sin 2x + 4\cot x + 2\sin^{-1}x + C$$

2. (2 points) Graph the integrand and use the area under the graph to evaluate the integral.

$$\int_0^3 \left(2 + \sqrt{9 - x^2}\right) dx$$

$$= \frac{1}{4}\pi(3)^2 + (2)(3)$$

Answer: $\frac{9\pi}{4} + 6$



3. (2 points) Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of

$$f(x) = \int_{1/x}^{2} \sin^3 t\, dt = -\int_2^{1/x} \sin^3 t\, dt = -\int_2^u \sin^2 t\, dt$$

$u = \frac{1}{x}$

Answer: $\frac{\sin^3\left(\frac{1}{x}\right)}{x^2}$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dt} = \frac{d}{du}\left(-\int_2^u \sin^3 t\, dt\right)\cdot\left(\frac{-1}{x^2}\right)$$

$$= \frac{\sin^3 u}{x^2} = \frac{\sin^3\left(\frac{1}{x}\right)}{x^2}$$

4. Evaluate. Write your answers in exact form.

a) (3 points) $\int_0^1 \left(\frac{3}{1+x^2} + \frac{1}{x+1}\right) dx$

$= \left[3\tan^{-1}x + \ln|x+1|\right]_0^1$

Answer: $\frac{3\pi}{4} + \ln 2$

$= 3\tan^{-1}1 + \ln|1+1| - (3\tan^{-1}0 + \ln|0+1|)$

$= 3\left(\frac{\pi}{4}\right) + \ln 2$

b) (3 points) $\int_0^{\pi/2} 2^{\cos x} \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$x=0$: $u = \cos 0 = 1$

$x = \frac{\pi}{2}$: $u = \cos\frac{\pi}{2} = 0$

Answer: $\frac{1}{\ln 2}$

$= -\int_1^0 2^u \, du$

$= \int_0^1 2^u \, du$

$= \left.\frac{2^u}{\ln 2}\right]_0^1$

$= \frac{2}{\ln 2} - \frac{1}{\ln 2}$

$= \frac{1}{\ln 2}$

c) (1 point) $\int_{-2}^{2} \underbrace{|x| \sin x}_{\text{odd}} \, dx$

Answer: 0

d) (3 points) $\int x^2 e^{-3x}\,dx$

Answer: $-\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}x e^{-3x} - \frac{2}{27}e^{-3x} + C$

$$\begin{array}{ccc} x^2 & + & e^{-3x} \\ 2x & - & -\frac{1}{3}e^{-3x} \\ 2 & + & \frac{1}{9}e^{-3x} \\ 0 & & -\frac{1}{27}e^{-3x} \end{array}$$

e) (3 points) $\int_0^{\pi/3} \sec^2 x\,dx$

$= \left[\tan x\right]_0^{\frac{\pi}{3}}$

$= \tan\frac{\pi}{3} - \tan 0$

$= \sqrt{3}$

Answer: $\sqrt{3}$