Math 180 - Test #2 Info and Review Exercises

Spring 2020, Prof. Beydler

Test Info

- Test #2 Date: Wednesday, April 8, 2020
- Will cover packets #7 through #16.
- You'll have from 9:30am to 2:00pm to finish the test.
- I'll post Test #2 as an assignment on Canvas on the test day at 9:30am.
  - If you have a printer, you can print the test out and write your solutions directly on it.
  - If you have a tablet, please feel free to use it to write directly into the PDF.
  - If you don't have a printer or tablet, feel free to use your own paper. Please put the test problems in order.
  - You can submit your test just like you do for your homework assignments. So, you'll attach the PDF under the Test #2 assignment.
- We're working on the honor system here: no notes, books, phones, or computers during the test.

Formulas and stuff

(Note: Know all of these except for the ones with 🌴 next to them, which I'll give you. This list is <u>not</u> meant to include <u>everything</u> you'll need to know on the test.)

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$(cf)' = cf'$

$(f \pm g)' = f' \pm g'$

$(fg)' = fg' + gf'$

$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ 🌴

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ 🌴

$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$ 🌴

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ 🌴

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ 🌴

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$ 🌴

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ 🌴

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ 🌴

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$ 🌴

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ 🌴

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ 🌴

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$ 🌴

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\tanh x = \frac{\sinh x}{\cosh x}$

$\coth x = \frac{\cosh x}{\sinh x}$ $\text{sech}\, x = \frac{1}{\cosh x}$ $\text{csch}\, x = \frac{1}{\sinh x}$

$\cosh^2 x - \sinh^2 x = 1$

Here are some helpful formulas to know for related rates problems:
Distance/rate/time formula: $d = rt$
Pythagorean Theorem: $a^2 + b^2 = c^2$ (or $(leg)^2 + (leg)^2 = (hypotenuse)^2$)
Area of rectangle: $A = lw$
Area of circle: $A = \pi r^2$
Area of triangle: $A = \frac{1}{2} bh$
Circumference of circle: $C = 2\pi r = \pi d$
How to get perimeter of any polygon (just add the lengths of the sides).
How to get the surface area of a 3-D surface (just add the areas of the faces/sides).
Volume of a box (also called a rectangular prism): $V = lwh$
Volume of circular cylinder: $V = \pi r^2 h$
Surface area of sphere: $S = 4\pi r^2$
Volume of sphere: $V = \frac{4}{3}\pi r^3$
Volume of cone: $V = \frac{1}{3}\pi r^2 h$

Newton's method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

L'Hospital's Rule
Suppose that $f(a) = g(a) = 0$, that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and submit them on Canvas on the test day, you can earn up to 3% extra credit towards your test! It is important to understand that these review exercises are <u>not guaranteed to cover all</u> of the potential problems on the test. Please review the notes and homework problems to fully prepare for the test.

1. Find $\frac{dy}{dx}$.

   a) $x^2 \sin y - \frac{3x}{y} = x^5$

   b) $x^2 e^y = \sqrt{y}$

c) $\cot(x + y) = 1 - \ln(y^2 + 3)$

2. Find an equation for the tangent line at the given point.

a) $x^2 + 4xy + y^2 = 13,\ (2, 1)$

b) $y = x^2 \cos^{-1}(3x + 3),\ (-1, 1)$

3. Use logarithmic differentiation to find $\frac{dy}{dx}$.

a) $y = \frac{e^{-5x^2+1} \cdot \sqrt[3]{x^2-4} \cdot \sin x}{x \cdot (x^3+1)^5 \cdot \sqrt{x+5}}$

b) $y = x^{\sec x}$

c) $y = (\csc x)^{1/x}$

4. The position of a particle is given by the equation $s(t) = t^3 - 12t + 3$ (where $t \geq 0$ is measured in seconds and s is measured in meters).
   a) What is the velocity after 1 second?

   b) When is the particle at rest?

   c) When is the particle moving in the positive direction?

   d) Sketch a diagram to represent the motion of the particle.

   e) Find the total distance traveled during the first 3 seconds.

   f) Find the acceleration at time t and after 5 seconds.

   g) When is the particle speeding up? When is it slowing down?

5. How fast is the volume of a sphere changing with respect to the radius when the radius is 3 inches?

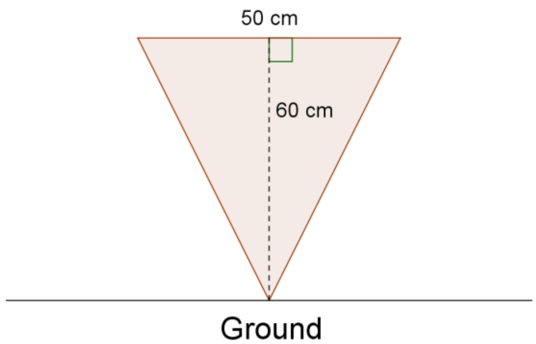
6. The mass of a thin rod from the left end to a point x mm to the right is $3\ln(x+2)$ grams. Find the linear density when x is 5 mm.

7. The width of a rectangle is increasing at a rate of 5 cm/s and its length is decreasing by 1 cm/s.
   a) When the width is 12 cm and the length is 16 cm, is the area of the rectangle increasing or decreasing? How fast is the area increasing or decreasing?

   b) Is the perimeter of the rectangle increasing or decreasing? How fast is the perimeter increasing or decreasing?

8. A ladder 20-ft long rests against a vertical wall. If the bottom of the ladder slides away at 2 ft/s, how fast is the top sliding down the wall when the bottom is 12 ft from the wall?

9. A trough is 5 meters long, and has cross sections that are isosceles triangles with base 50 cm and height 60 cm (as shown below). If the trough is being filled with water at a rate of 300 cm^3/min, how fast is the water level rising when the water is 20 cm deep?



10. Prove that the derivative of $y = \cot x$ is $\frac{dy}{dx} = -\csc^2 x$ by using the derivatives of $\sin x$ and $\cos x$.

11. Prove that the derivative of $y = \ln x$ is $\frac{dy}{dx} = \frac{1}{x}$.

12. Prove that the derivative of $y = \cos^{-1} x$ is $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$.

13. Find the differential (dy) of $y = e^{-x} \cos x$.

14. Find the differential (dy) of $y = \sec^{-1}(3x)$.

15. Use a linear approximation (or differentials) to estimate $\frac{1}{(2.999)^5}$.

16. The radius of a circle was measured to be 5 ft with a possible error of 0.1 ft.

a) Use differentials to estimate the maximum error in the calculated area of the circle. What is the relative error? What is the percentage error?

b) Use differentials to estimate the maximum error in the calculated circumference of the circle. What is the relative error? What is the percentage error?

17. Use Newton's method to estimate the positive root of $\frac{1}{x} = 1 + x^3$ correct to six decimal places. Start with $x_1 = 0.8$.

18. Find the following limits.

a) $\lim\limits_{x\to 0} \frac{\sin x - x}{x^2}$

b) $\lim_{x \to 0} \frac{e^x - e^{-x} - 2\sin x}{3x^3}$

c) $\lim_{x \to \infty} x\left(e^{1/x} - 1\right)$

d) $\lim_{x \to 0^+} \sin x \ln(\sin x)$

e) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

f) $\lim_{x \to 0^+} x^{x^2}$

g) $\lim_{x \to \infty} (x + 1)^{e^{-x}}$

19. Let $f(x) = \frac{x}{x^2+1}$.

a) Find the domain of f.

b) Find the x-intercept(s) and y-intercept of f (if any).

c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).

d) Find f' and f'', and determine where each are 0 and/or do not exist (DNE).

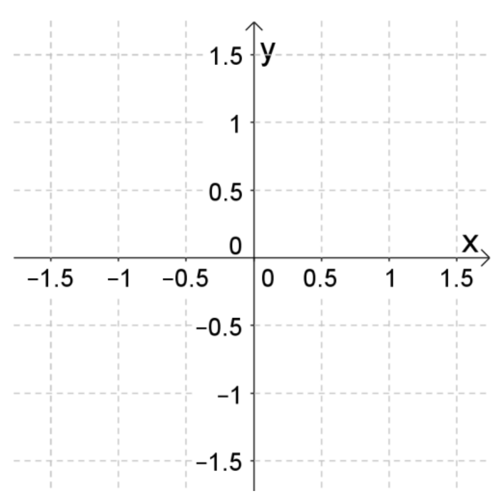
e) Do a sign analysis on f' and f''.

f) Find the intervals on which f is increasing and decreasing.

g) Find the intervals on which f is concave up and concave down.

h) Find all local maxima, local minima, and inflection points of f.

i) Sketch the graph of f.



20. Let $f(x) = x\sqrt{1-x^2}$.

a) Find the domain of f.

b) Find the x-intercept(s) and y-intercept of f (if any).

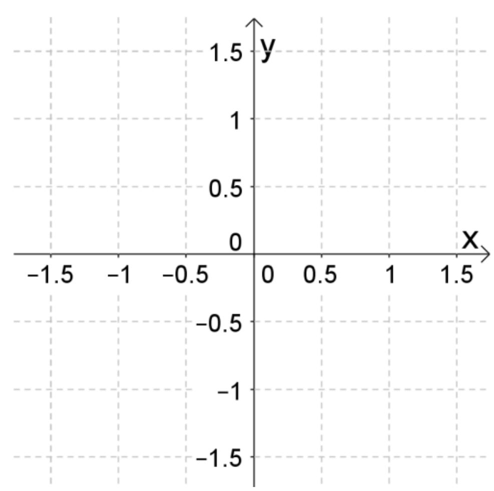
c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).

d) Find f' and f'', and determine where each are 0 and/or do not exist (DNE).

e) Do a sign analysis on f' and f''.

f) Find the intervals on which f is increasing and decreasing.

g) Find the intervals on which f is concave up and concave down.



h) Find all local maxima, local minima, and inflection points of f.

i) Sketch the graph of f.

21. Let $f(x) = 4x^{1/3} - x^{4/3}$.

a) Find the domain of f.

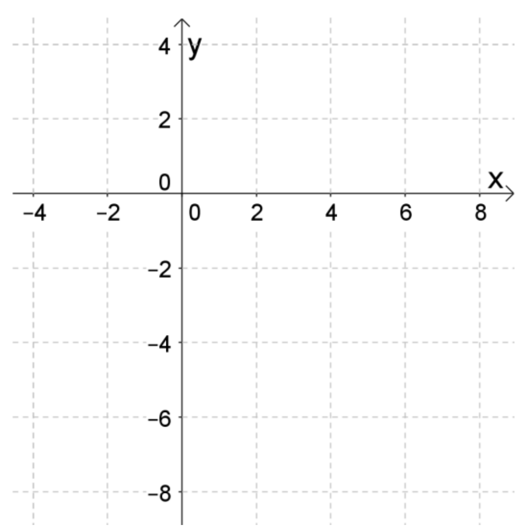
b) Find the x-intercept(s) and y-intercept of f (if any).

c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).

d) Find f' and f'', and determine where each are 0 and/or do not exist (DNE).

e) Do a sign analysis on f' and f''.

f) Find the intervals on which f is increasing and decreasing.



g) Find the intervals on which f is concave up and concave down.

h) Find all local maxima, local minima, and inflection points of f.

i) Sketch the graph of f.

22. Let $f(x) = e^x(x^2 - 3)$.

a) Find the domain of f.

b) Find the x-intercept(s) and y-intercept of f (if any).

c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).

d) Find f' and f'', and determine where each are 0 and/or do not exist (DNE).

e) Do a sign analysis on f' and f''.

f) Find the intervals on which f is increasing and decreasing.

g) Find the intervals on which f is concave up and concave down.

h) Find all local maxima, local minima, and inflection points of f.

i) Sketch the graph of f.