Test #2 (Part 2 – Calculator Okay) Name: ____________________

Math 180, Prof. Beydler Wednesday, October 24, 2018

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. The position of a particle is given by the equation $s(t) = t^3 - 9t^2 + 24t + 1$ (where $t \geq 0$ is measured in seconds and s is measured in meters).

a) (1 point) When is the particle at rest?

$$v(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8)$$
$$= 3(t-2)(t-4)$$
$$t = 2 \qquad t = 4$$

Answer: $t = 2$ sec, $t = 4$ sec

b) (1 point) When is the particle moving in the positive direction?

t	0		2		4	
$v(t)$	+	+	0	−	0	+

Answer: $0 \leq t < 2$, $t > 4$

c) (2 points) Find the total distance traveled during the first 6 seconds. Be sure to include units for your answer.

$$|s(2) - s(0)| = |21 - 1| = 20$$
$$|s(4) - s(2)| = |17 - 21| = 4$$
$$|s(6) - s(4)| = |37 - 17| = 20$$

Answer: 44 m

d) (1 point) Find the acceleration after 2 seconds. Be sure to include units for your answer.

$$a(t) = 6t - 18 = 6(t-3)$$
$$a(2) = 6(2) - 18 = -6$$

Answer: $-6\ m/s^2$

e) (3 points) When is the particle speeding up? When is it slowing down?

t	0		2		3		4	
v	+	+	0	−	−	−	0	+
a	−	−	−	−	0	+	+	+

Speeding up: $2 < t < 3$, $t > 4$

Slowing down: $0 \leq t < 2$, $3 < t < 4$

2. (1 point) The mass of a thin wire from the left end to a point x inches to the right is e^{2x-1} ounces. Find the linear density when x is 3 inches. Approximate your answer to the nearest 3 decimal places. Be sure to write units for your answer.

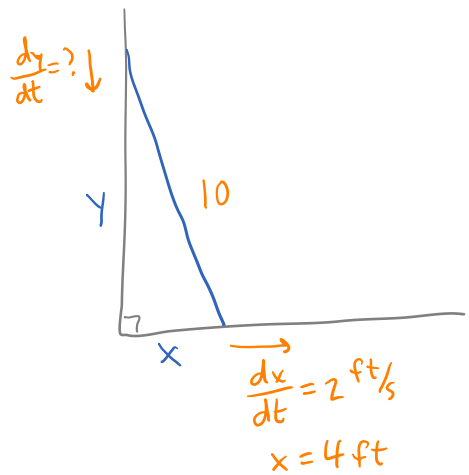
$m'(x) = 2e^{2x-1}$

$m'(3) = 2e^{2(3)-1} = 2e^5$

Answer: 296.826 oz/in

3. (4 points) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away at 2 ft/s, how fast is the top sliding down the wall when the bottom is 4 ft from the wall? Be sure to write units for your answer.

Answer: $\frac{4}{\sqrt{21}}$ ft/s



$x^2 + y^2 = 10^2$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y} = \frac{-(4)(2)}{2\sqrt{21}} = \frac{-4}{\sqrt{21}}$

$4^2 + y^2 = 10^2$

$y = \sqrt{84} = 2\sqrt{21}$

4. (3 points) Use a <u>linear approximation</u> (or <u>differentials</u>) to estimate $\sqrt[4]{15.99}$ to 5 decimal places. Be sure to show your work.

$\sqrt[4]{15.99} \approx$ 1.99969

$f(x) = \sqrt[4]{x} = x^{1/4}$

$f'(x) = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$

$L(x) = f(16) + f'(16)(x-16)$

$= \sqrt[4]{16} + \frac{1}{4(16)^{3/4}}(x-16)$

$= 2 + \frac{1}{32}(x-16)$

$= \frac{1}{32}x + \frac{3}{2}$

$L(15.99) = \frac{1}{32}(15.99) + \frac{3}{2}$

5. Let $f(x) = 4x^{1/3} - x^{4/3}$.

a) (1 point) Find the domain of f.

Domain: $(-\infty, \infty)$

b) (1 point) Find the x-intercept(s) and y-intercept of f (if none, write "none").

$$4x^{1/3} - x^{4/3} = 0$$
$$x^{1/3}(4-x) = 0$$

x-intercept(s): $0, 4$ y-intercept: 0

c) (4 points) Find f' and f'', and determine where each are 0 and/or do not exist (DNE). If nowhere, write "nowhere."

$$f'(x) = \frac{4}{3}x^{-2/3} - \frac{4}{3}x^{1/3} = \frac{4}{3}x^{-2/3}(1-x) = \frac{4(1-x)}{3x^{2/3}}$$

$f'=0$:
$1-x=0$
$x=1$

f' DNE:
$x=0$

$$f''(x) = -\frac{8}{9}x^{-5/3} - \frac{4}{9}x^{-2/3} = -\frac{4}{9}x^{-5/3}(2+x) = \frac{-4(2+x)}{9x^{5/3}}$$

$f''=0$:
$2+x=0$
$x=-2$

f'' DNE:
$x=0$

$f'(x) = 0$ when $x =$ 1

$f'(x)$ DNE when $x =$ 0

$f''(x) = 0$ when $x =$ -2

$f''(x)$ DNE when $x =$ 0

d) (2 points) Do a sign analysis on f' and f''.

		-2		0		1	
f'	$+$	$+$	$+$	DNE	$+$	0	$-$
f''	$-$	0	$+$	DNE	$-$	$-$	$-$

e) (1 point) Find the intervals on which f is increasing and decreasing.

Increasing: $(-\infty, 1)$ Decreasing: $(1, \infty)$

f) (1 point) Find the intervals on which f is concave up and concave down.

Concave up: $(-2, 0)$ Concave down: $(-\infty, -2), (0, \infty)$

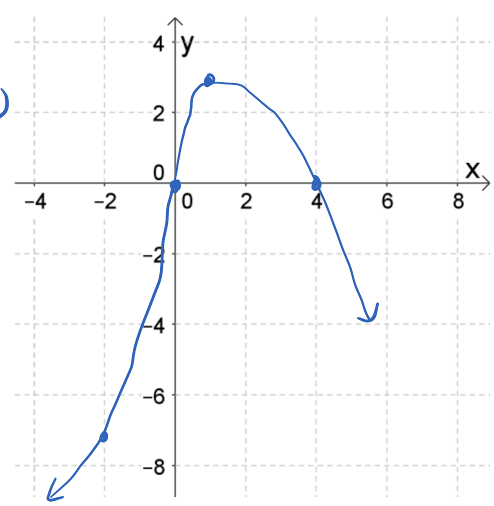
g) (1 point) Find all local maxima, local minima, and inflection points of f (if any). Be sure to identify which is a max/min/inflection point.

Max: $(1, 3)$

IP: $(-2, 6\sqrt[3]{-2})$ -7.6

IP: $(0, 0)$

h) (2 points) Sketch the graph of f.



6. (3 points) The radius of a sphere was measured to be 2 inches with a possible error of 0.01 inches. Use differentials to estimate the maximum error in the calculated <u>surface area</u> of the sphere. What is the percentage error? Be sure to write units for your answer.

Estimated maximum error: 0.50265 in^2

Estimated percentage error: 1%

$$S = 4\pi r^2$$

$$dS = 8\pi r\,dr$$

$$= 8\pi(2)(0.01)$$

$$\approx 0.50265$$

$$\frac{dS}{S} \approx \frac{0.50265}{4\pi(2)^2} \approx 0.01 \xrightarrow{\times 100\%} 1\%$$

7. (2 points) Use Newton's method to estimate the positive root of $\cos x = x^2$ correct to six decimal places. Start with $x_1 = 1$.

Answer: 0.824132

$$\underbrace{\cos x - x^2}_{f(x)} = 0 \qquad f'(x) = -\sin x - 2x$$

$$x_2 = 1 - \frac{\cos 1 - 1}{-\sin 1 - 2(1)} \approx 0.838218$$

$$x_3 \approx 0.824242$$

$$x_4 \approx 0.824132$$

$$x_5 \approx 0.824132$$

Note: Be sure to double check your work. And remember to turn in your homework and extra credit! ☺

Here are some formulas that I promised to give:

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$