_____ / 50 total points (both parts)

Test #2 (Part 1 – No Calculator)

Name: ______________________________

Math 180, Prof. Beydler

Wednesday, October 24, 2018

Directions: Show all work. No calculator, books, or notes. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. When you're finished with Part 1, please turn it in, take a bathroom break, get your calculator out, and start Part 2. Good luck!

1. (4 points) Find an equation for the tangent line of $x^2y + e^{x+y} = 1$ at $(0,0)$.

$$\frac{d}{dx}(x^2y) + \frac{d}{dx}(e^{x+y}) = \frac{d}{dx}(1)$$

$$x^2 \cdot \frac{dy}{dx} + 2xy + e^{x+y}\left(1 + \frac{dy}{dx}\right) = 0$$

$$x^2\frac{dy}{dx} + 2xy + e^{x+y} + e^{x+y}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x^2 + e^{x+y}) = -2xy - e^{x+y}$$

$$\frac{dy}{dx} = \frac{-2xy - e^{x+y}}{x^2 + e^{x+y}}$$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{0 - e^0}{0 + e^0} = -1$$

Answer: $y = -x$

$$y - 0 = -1(x - 0)$$

2. (3 points) Use <u>logarithmic differentiation</u> to find $\frac{dy}{dx}$ given that $y = \frac{\sqrt[4]{x}\cdot(2x^3+1)^2}{e^{\tan^{-1}x}}$.

$\frac{dy}{dx} =$ $y\left(\frac{1}{4x} + \frac{12x^2}{2x^3+1} - \frac{1}{1+x^2}\right)$

$$\ln y = \ln \frac{x^{1/4}(2x^3+1)^2}{e^{\tan^{-1}x}}$$

$$= \frac{1}{4}\ln x + 2\ln(2x^3+1) - \tan^{-1}x$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{4x} + 2\cdot\frac{1}{2x^3+1}\cdot 6x^2 - \frac{1}{1+x^2}$$

3. (3 points) Prove that the derivative of $y = \csc x$ is $\frac{dy}{dx} = -\csc x \cot x$ by using the derivatives of $\sin x$ and/or $\cos x$.

$y = \csc x = \frac{1}{\sin x}$

$$\frac{dy}{dx} = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = \frac{-\cos x}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$$

4. Find the following limits. Be sure to show your work.

a) (3 points) $\lim_{x\to 0^+} \frac{1-\cos x}{x^2}$

$$= \lim_{x\to 0^+} \frac{\sin x}{2x}$$

$$= \lim_{x\to 0^+} \frac{\cos x}{2}$$

Answer: $\frac{1}{2}$

b) (3 points) $\lim_{x\to 0^+} (1-2x)^{1/x}$

$$\lim_{x\to 0^+} \ln(1-2x)^{1/x} = \lim_{x\to 0^+} \frac{\ln(1-2x)}{x}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{1-2x}\cdot(-2)}{1}$$

$$= \lim_{x\to 0^+} \frac{-2}{1-2x}$$

$$= -2$$

So, $\lim_{x\to 0^+} (1-2x)^{1/x}$

$$= \lim_{x\to 0^+} e^{\ln(1-2x)^{1/x}}$$

$$= e^{-2}$$

Answer: $\frac{1}{e^2}$