_____ / 50 total points

Test #2

Name: ________________________________

Math 180, Prof. Beydler

Wednesday, May 6, 2020

Directions: We're working on the honor system here: no notes, books, phones, or computers during the test (except for using a computer to write your answers). Also, no getting help from other people. You may e-mail me to ask for clarification about any problem. **Show all work**. A **scientific calculator** is allowed. Write your answers in the indicated places, or box your answers. Good luck!

1. The position of a particle is given by the equation $s(t) = t^3 - 12t^2 + 21t$ (where $t \geq 0$ is measured in seconds and s is measured in meters).

   a) (1 point) When is the particle at rest?

   Answer: ______________________

   b) (1 point) When is the particle moving in the positive direction?

   Answer: ________________________

   c) (2 points) Find the total distance traveled during the first 8 seconds. Be sure to include units for your answer.

   Answer: ____________

   d) (1 point) Find the acceleration after 5 seconds. Be sure to include units for your answer.

   Answer: ______________

   e) (3 points) When is the particle speeding up? When is it slowing down?

   Speeding up: _____________________________

   Slowing down: ____________________________

2. (1 point) The mass of a thin wire from the left end to a point x inches to the right is e^{3x+1} ounces. Find the linear density when x is 2 inches. Approximate your answer to the nearest 3 decimal places. Be sure to write units for your answer.

Answer: ____________________

3. (4 points) Find an equation for the tangent line of $\tan(2x + y) + y^2 = 0$ at $(0, 0)$.

Answer: ______________________

4. (3 points) Use **<u>logarithmic differentiation</u>** to find $\frac{dy}{dx}$ given that $y = \frac{(5x^2-3)^4}{\sqrt[3]{x} \cdot e^{\tan^{-1} x}}$

$\frac{dy}{dx} =$ ____

5. (4 points) Water runs into a conical tank at the rate of 5 ft^3/min. The tank stands point down and has a height of 8 ft and a base radius of 2 ft. How fast is the water level rising when the water is 3 ft deep? Be sure to write units for your answer.

Answer: ________________

6. (3 points) Prove that the derivative of $y = \ln x$ is $\frac{dy}{dx} = \frac{1}{x}$ by using the derivative of e^x.

7. (3 points) Use a <u>linear approximation</u> (or <u>differentials</u>) to estimate $\sqrt[3]{8.01}$ to 5 decimal places. Be sure to show your work.

$\sqrt[3]{8.01} \approx$ ____________

8. (3 points) The radius of a sphere was measured to be 3 inches with a possible error of 0.01 inches. Use differentials to estimate the maximum error in the calculated surface area of the sphere. Write your answer to 3 decimal places. What is the percentage error? Be sure to write units for your answer.

Estimated maximum error: ____________

Estimated percentage error: ___________

9. (2 points) Use Newton's method to estimate the positive root of $\cos x = x^2$ correct to six decimal places. Start with $x_1 = 1$.

Answer: ______________________

10. Find the following limits. Be sure to show your work.

a) (3 points) $\lim_{x \to 0^+} \frac{1 - \cos x}{x^2}$

Answer: __________

b) (3 points) $\lim_{x \to \infty} (2x - 1)^{e^{-x}}$

Answer: __________

11. Let $f(x) = 2x^{5/3} - 5x^{2/3}$.

a) (1 point) Find the domain of f.

Domain: ____________________

b) (1 point) Find the x-intercept(s) and y-intercept of f (if none, write "none").

x-intercept(s): __________ y-intercept: ______

c) (4 points) Find f' and f'', and determine where each are 0 and/or do not exist (DNE). If nowhere, write "nowhere."

$f'(x) = 0$ when $x =$ __________

$f'(x)$ DNE when $x =$ __________

$f''(x) = 0$ when $x =$ __________

$f''(x)$ DNE when $x =$ __________

d) (2 points) Do a sign analysis on f' and f''.

e) (1 point) Find the intervals on which f is increasing and decreasing.

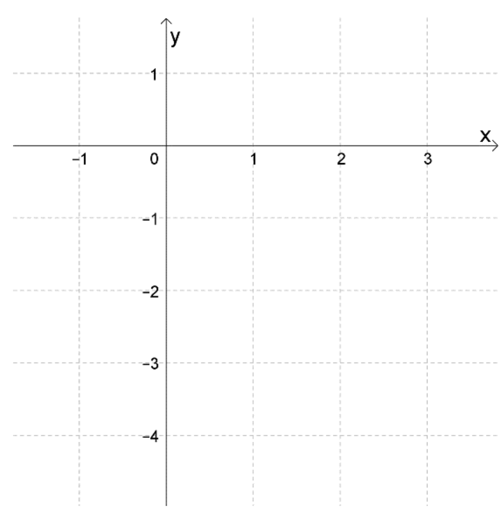
Increasing: ____________________________ Decreasing: ______________________

f) (1 point) Find the intervals on which f is concave up and concave down.

Concave up: ___________________ Concave down: ___________________

g) (1 point) Find all local maxima, local minima, and inflection points of f (if any). Be sure to identify which is a max/min/inflection point.

h) (2 points) Sketch the graph of f.



Note: Be sure to double check your work. And remember to turn in your homework and extra credit! ☺

Here are some formulas that I promised to give:

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$