Test #1 Review Exercise Answers

1.

$\lim_{x\to 0^-} f(x) = 1$ $\lim_{x\to 0^+} f(x) = 2$ $\lim_{x\to 0} f(x)\ DNE$

$\lim_{x\to 1^-} f(x) = 1$ $\lim_{x\to 1^+} f(x) = 1$ $\lim_{x\to 1} f(x) = 1$

$\lim_{x\to 2^-} f(x) = 2$ $\lim_{x\to 2^+} f(x) = 2$ $\lim_{x\to 2} f(x) = 2$

$\lim_{x\to 3^-} f(x) = 5$ $\lim_{x\to 3^+} f(x) = 3$ $\lim_{x\to 3} f(x)\ DNE$

$\lim_{x\to 5^-} f(x) = 3$ $\lim_{x\to 5^+} f(x) = 3$ $\lim_{x\to 5} f(x) = 3$

$\lim_{x\to 6^-} f(x) = 0$ $\lim_{x\to 6^+} f(x) = 0$ $\lim_{x\to 6} f(x) = 0$

$\lim_{x\to 7^-} f(x) = \infty$ $\lim_{x\to 7^+} f(x) = -\infty$ $\lim_{x\to 7} f(x)\ DNE$

$\lim_{x\to \infty} f(x) = -1$ $\lim_{x\to -\infty} f(x) = \infty$

2.

a) discontinuous because $\lim_{x\to 1} f(x) = 1$ but $f(1) = 2$ (so $\lim_{x\to 1} f(x) \neq f(1)$)
b) discontinuous because $f(5)$ is undefined
c) continuous because $\lim_{x\to 6} f(x) = 0 = f(6)$
d) no because $\lim_{x\to 1^-} f(x) = 1$ but $f(1) = 2$ (so $\lim_{x\to 1^-} f(x) \neq f(1)$)
e) yes because $\lim_{x\to 3^+} f(x) = 3 = f(3)$
f) no because $f(7)$ is undefined (or you could say "because $\lim_{x\to 7^-} f(x) = \infty$")
g) no
h) yes
i) -1
j) DNE
k) DNE
l) 0
m) -3
n) $y - 3 = -(x + 2)$ (or $y = -x + 1$)

3.

a) 2
b) ∞
c) DNE
d) -7
e) $\frac{1}{6}$
f) $\frac{1}{2}$
g) $\frac{1}{9}$
h) $-\frac{\pi}{2}$
i) ∞
j) 0
k) $\frac{2}{7}$
l) 0
m) ∞
n) $\frac{1}{\sqrt{3}}$
o) 0
p) ∞

4. First off, $-1 \leq \sin\left(\frac{5}{x-1}+3\right) \leq 1$. Multiplying by $(x-1)^2$, we have $-(x-1)^2 \leq \sin\left(\frac{5}{x-1}+3\right) \leq (x-1)^2$. Since $\lim_{x\to 1}(-(x-1)^2) = \lim_{x\to 1}(x-1)^2 = 0$, by the Squeeze Theorem $\lim_{x\to 1}(x-1)^2 \sin\left(\frac{5}{x-1}+3\right) = 0$. ■

5. $\lim_{x\to -2^-}(2x^2-3) = 5$ but $\lim_{x\to -2^+}\frac{5}{x+2} = \infty$, so $\lim_{x\to -2} f(x)$ DNE. Thus, f is discontinuous at $x = -2$.

6. $f(1)$ is undefined. Thus, f is discontinuous at $x = 1$.

7. $-\frac{6}{5}$

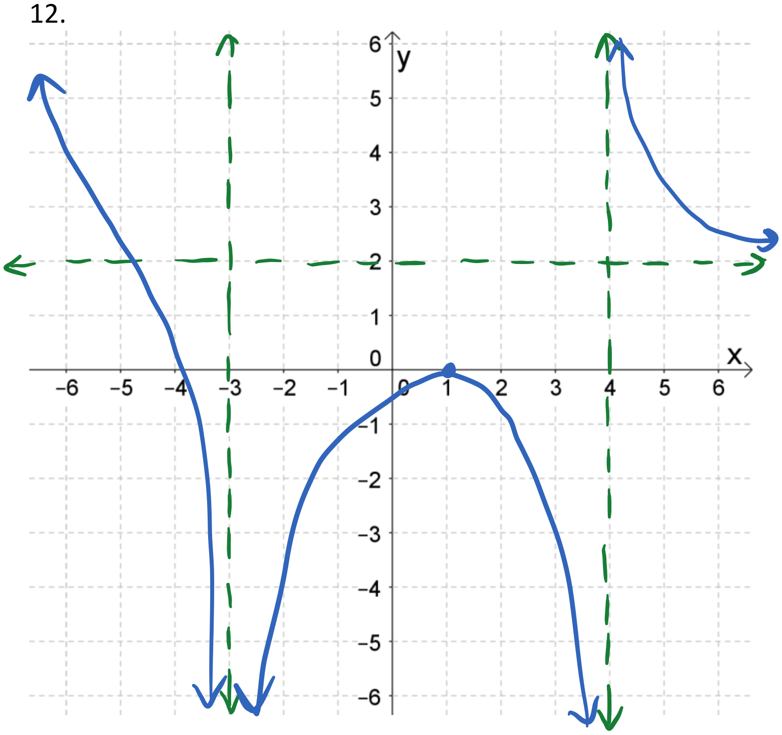
8. $c = 3$

9. True

10. False (It should be $\lim_{x\to a} f(x) = f(a)$.)

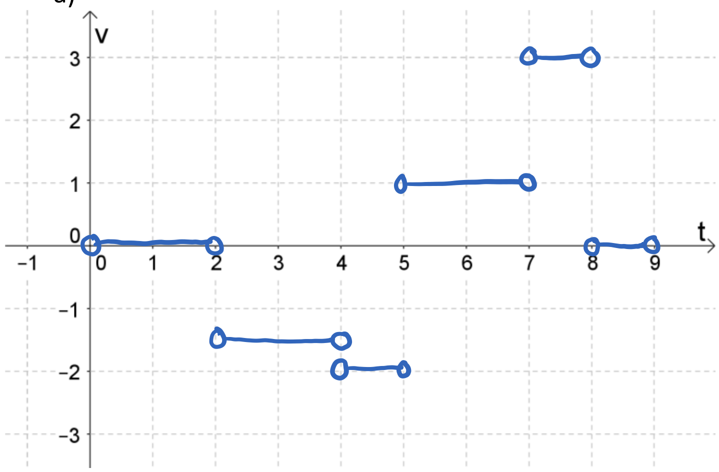
11. First, subtract the $\sin 2x$ to get $x - 3 - \sin 2x = 0$. Then let $f(x) = x - 3 - \sin 2x$. Since f is continuous, $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} - 3 < 0$, and $f(\pi) = \pi - 3 > 0$, we know by the Intermediate Value Theorem that there must be some x in the interval $\left(\frac{\pi}{2}, \pi\right)$ such that $f(x) = 0$. At that x, we have a root of $x - 3 = \sin 2x$. ■

12.



13.

a)



b) -1.5 m/s

c) DNE

d) 3 m/s

14.

a) $4x - 3$

b) $\frac{3}{2\sqrt{3x+1}}$

c) $-\frac{2}{(x+5)^2}$

15.

a) $f'(x) = -\frac{4}{\sqrt[3]{x}} - \frac{8}{x^5} - 2.6x^{0.3} - \frac{2}{3x\sqrt{x}} - 17e^x$

b) $f'(x) = \frac{7}{2}x^2\sqrt{x} - 1 - \frac{1}{x\sqrt{x}}$

c) $f'(x) = x^3e^x + 3x^2e^x - \frac{2}{x^2}$

d) $f'(x) = \frac{x^2e^x - 3xe^x + 4e^x - 4x + 2}{(x^2 - x + 3)^2}$

e) $f'(x) = -3\sin x - 2x^3\cos x - 6x^2\sin x$

f) $f'(x) = -\sec x \csc x \cot x + \csc x \sec x \tan x$

g) $f'(x) = \frac{-\tan x \csc^2 x - 6x\tan x - \cot x \sec^2 x + 3x^2 \sec^2 x}{\tan^2 x}$

h) $f'(x) = \frac{xe^x\cos x + xe^x \sin x - 3e^x \sin x}{x^4}$ (or $\frac{e^x(x\cos x + x\sin x - 3\sin x)}{x^4}$)

i) $\frac{\cos\sqrt{2x+3}}{\sqrt{2x+3}\sin\sqrt{2x+3}}$ (or $\frac{\cot\sqrt{2x+3}}{\sqrt{2x+3}}$)

j) $3^{x^2 - \cos(\log_2 x)} \cdot \ln 3 \cdot \left(2x + \sin(\log_2 x) \cdot \frac{1}{x\ln 2}\right)$

k) $\left(\ln\sqrt{x} + x^5\right) \cdot \sec^2(e^{3x+1}) \cdot e^{3x+1} \cdot 3 + \tan(e^{3x+1}) \cdot \left(\frac{1}{2x} + 5x^4\right)$

l) $\frac{3(x^2+3)\sin^2 x \cos x \ln(x^2+3) - 2x\sin^3 x}{(x^2+3)(\ln(x^2+3))^2}$

m) $\frac{2x\tan^{-1}x - 1}{(\tan^{-1}x)^2}$

n) $\frac{1}{3\sqrt[3]{(x^2\sin^{-1}x)^2}}\left(\frac{x^2}{\sqrt{1-x^2}} + 2x\sin^{-1}x\right)$

o) $\sinh(\sinh(\tanh x)) \cdot \cosh(\tanh x) \cdot \operatorname{sech}^2 x$

16.

a) $y + 2 = -2(x - 0)$ or $y = -2x - 2$

b) $y - 1 = 2\left(x - \frac{\pi}{4}\right)$ or $y = 2x - \frac{\pi}{2} + 1$

c) $y = 2x$

17.

a) $y'' = 4e^x + 12x + \frac{2}{x^3}$

b) $y'' = -x^2 \cos x - 4x \sin x + 2 \cos x$