Name: ______________________________

Math 180 - Test #1 Info and Review Exercises

Spring 2020, Prof. Beydler

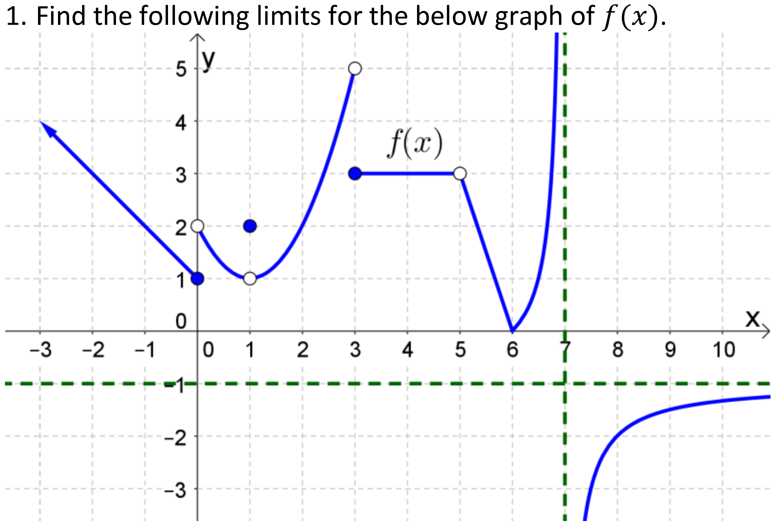
Test Info

- Test #1 Date: Wednesday, April 8, 2020
- Will cover packets #1 through #6.
- You'll have from 9:30am to 2:00pm to finish the test.
- I'll post Test #1 as an assignment on Canvas on the test day at 9:30am.
    - If you have a printer, you can print the test out and write your solutions directly on it.
    - If you have a tablet, please feel free to use it to write directly into the PDF.
    - If you don't have a printer or tablet, feel free to use your own paper. Please put the test problems in order.
    - You can submit your test just like you do for your homework assignments. So, you'll attach the PDF under the Test #1 assignment.
- We're working on the honor system here: no notes, books, phones, or computers during the test.

Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and submit them on Canvas on the test day, you can earn up to 3% extra credit towards your test! It is important to understand that these review exercises are <u>not guaranteed to cover all</u> of the potential problems on the test. Please review the notes and homework problems to fully prepare for the test.

1. Find the following limits for the below graph of $f(x)$.



$\lim_{x\to 0^-} f(x)$	$\lim_{x\to 0^+} f(x)$	$\lim_{x\to 0} f(x)$
$\lim_{x\to 1^-} f(x)$	$\lim_{x\to 1^+} f(x)$	$\lim_{x\to 1} f(x)$
$\lim_{x\to 2^-} f(x)$	$\lim_{x\to 2^+} f(x)$	$\lim_{x\to 2} f(x)$
$\lim_{x\to 3^-} f(x)$	$\lim_{x\to 3^+} f(x)$	$\lim_{x\to 3} f(x)$
$\lim_{x\to 5^-} f(x)$	$\lim_{x\to 5^+} f(x)$	$\lim_{x\to 5} f(x)$
$\lim_{x\to 6^-} f(x)$	$\lim_{x\to 6^+} f(x)$	$\lim_{x\to 6} f(x)$
$\lim_{x\to 7^-} f(x)$	$\lim_{x\to 7^+} f(x)$	$\lim_{x\to 7} f(x)$
$\lim_{x\to \infty} f(x)$	$\lim_{x\to -\infty} f(x)$	

2. Using the graph of $f(x)$ from the previous question, answer the following.
   a) Is f continuous or discontinuous at $x = 1$? Why?
   b) Is f continuous or discontinuous at $x = 5$? Why?
   c) Is f continuous or discontinuous at $x = 6$? Why?
   d) Is f continuous from the left at $x = 1$? Why or why not?
   e) Is f continuous from the right at $x = 3$? Why or why not?
   f) Is f continuous from the left at $x = 7$? Why or why not?
   g) Is f differentiable at $x = 6$?
   h) Is f differentiable at $x = -2$?
   i) Find $f'(-5)$.
   j) Find $f'(1)$.
   k) Find $f'(3)$.
   l) Find $f'(4)$.
   m) Find $f'(5.5)$.
   n) Find an equation of the tangent line at $x = -2$.

3. Find the following limits.
   a) $\lim_{x\to-1} \frac{x^2-2x-3}{x^2-1}$

   b) $\lim_{x\to-5^+} \frac{3-x}{x+5}$

   c) $\lim_{x\to-2} \frac{3x+4}{x+2}$

   d) $\lim_{x\to2} \frac{3x^2-5x-2}{x^2-5x+6}$

e) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

f) $\lim_{x\to 1^-} \frac{x-\sqrt{x}}{x-1}$

g) $\lim_{x\to 0} \frac{\frac{1}{x-3}+\frac{1}{3}}{x(x-1)}$

h) $\lim_{x\to\infty} \tan^{-1}(2 - x^2)$

i) $\lim_{x\to 0^+} e^{2-\ln x}$

j) $\lim_{x\to 1^-} \tan\left(e^{\csc(x-1)}\right)$

k) $\lim_{x\to -\infty} \frac{2x^5-x+1}{7x^5+4x^4-3}$

l) $\lim_{x\to\infty} \frac{e^x-2}{1+e^{2x}}$

m) $\lim_{x\to -\infty} \frac{x^3-21x}{5x+2}$

n) $\lim_{x\to\infty} \frac{x+2}{\sqrt{3x^2-5x}}$

o) $\lim\limits_{x\to-\infty}\left(2x+\sqrt{4x^2+1}\right)$

p) $\lim\limits_{x\to0^+}(\cot x-\ln x)$

4. Use the Squeeze Theorem to prove that $\lim\limits_{x\to1}(x-1)^2\sin\left(\frac{5}{x-1}+3\right)=0$.

5. Explain why $f(x)=\begin{cases}2x^2-3 & \text{if } x\le -2\\ \frac{5}{x+2} & \text{if } x>-2\end{cases}$ is discontinuous at $x=-2$.

6. Explain why $f(x)=\begin{cases}x^2+2 & \text{if } x<1\\ \sqrt{x+8} & \text{if } x>1\end{cases}$ is discontinuous at $x=1$.

7. How would you define $f(-1)$ in a way that makes $f(x) = \frac{5+4x-x^2}{2x^2-x-3}$ continuous at $x = -1$?

8. Find all values of c such that $f(x) = \begin{cases} x^2 - c & \text{if } x \leq 2 \\ \sin\frac{\pi}{4}x & \text{if } x > 2 \end{cases}$ is continuous on $(-\infty, \infty)$.

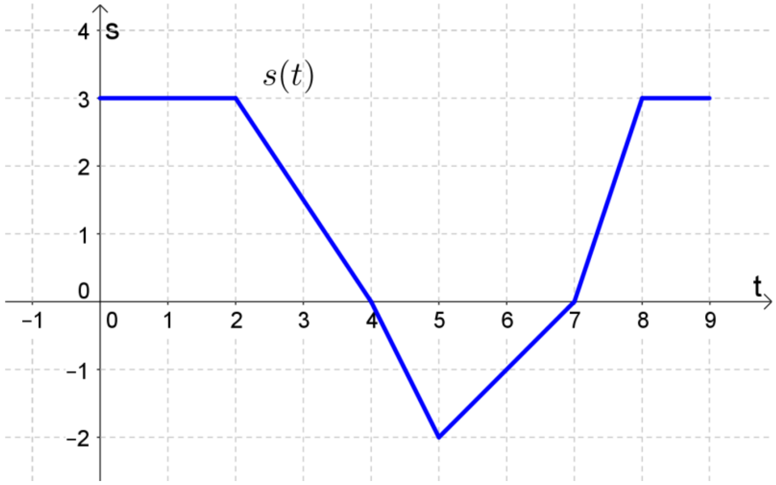
9. True or false: If $\lim_{x\to 3^-} f(x) = -\infty$, then f has a vertical asymptote $x = 3$.

10. True or false: A function is continuous at $x = a$ if $\lim_{h\to 0} f(x) = f(a)$.

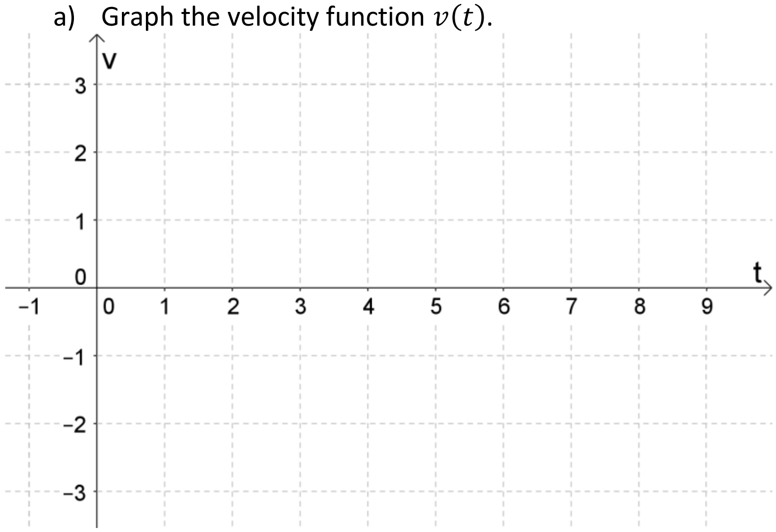
11. Use the Intermediate Value Theorem to show that there is a root of $x - 3 = \sin 2x$ in the interval $\left(\frac{\pi}{2}, \pi\right)$.

12. Sketch the graph of an example of a function f that satisfies $\lim_{x\to -3} f(x) = -\infty$, $\lim_{x\to 4^-} f(x) = -\infty$, $\lim_{x\to 4^+} f(x) = \infty$, $\lim_{x\to -\infty} f(x) = \infty$, $\lim_{x\to \infty} f(x) = 2$, and $f(1) = 0$.

13. Suppose an ant moves back and forth along a line, and that the position of the ant over time is given by the function $s(t)$ below. Use meters for s and seconds for t.



a) Graph the velocity function $v(t)$.



b) Find $s'(3)$.

c) Find $s'(5)$.

d) Find $v(7.5)$.

14. Find the derivatives of the following functions <u>using the limit definition</u>.

a) $f(x) = 2x^2 - 3x$

b) $f(x) = \sqrt{3x+1}$

c) $f(x) = \frac{2}{x+5}$

15. Differentiate the following functions.

a) $f(x) = 3\pi^5 - 6\sqrt[3]{x^2} + \frac{2}{x^4} - 2x^{1.3} + 5e^2 + \frac{4}{3\sqrt{x}} - 17e^x$

b) $f(x) = \frac{x^4 - x\sqrt{x} + 2}{\sqrt{x}}$

c) $f(x) = x^3 e^x + \frac{2}{x}$

d) $f(x) = \frac{e^x+2}{x^2-x+3}$

e) $f(x) = 3\cos x - 2x^3 \sin x$

f) $f(x) = \sec x \csc x$

g) $f(x) = \frac{\cot x - 3x^2}{\tan x}$

h) $f(x) = \frac{e^x \sin x}{x^3}$

i) $f(x) = \ln\left(\sin\sqrt{2x+3}\right)$

j) $y = 3^{x^2 - \cos(\log_2 x)}$

k) $f(x) = \left(\ln\sqrt{x} + x^5\right)\tan(e^{3x+1})$

l) $y = \frac{\sin^3 x}{\ln(x^2+3)}$

m) $f(x) = \frac{x^2+1}{\tan^{-1} x}$

n) $y = \sqrt[3]{x^2 \sin^{-1} x}$

o) $f(x) = \cosh(\sinh(\tanh x))$

16. Find an equation for the tangent line at the given point.

a) $y = x^3 - 2e^x,\ (0, -2)$

b) $y = \tan x,\ \left(\frac{\pi}{4}, 1\right)$

c) $y = 2\sin x \cos x - x^2,\ (0, 0)$

17. Find the second derivatives of the following functions.

a) $y = 4e^x + 2x^3 - x + \frac{1}{x}$

b) $y = x^2 \cos x$