# Test #1 (Part 2 – Calculator Okay)

Math 180, Prof. Beydler

Name: ______________________________

Wednesday, September 26, 2018

**Directions:** Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. Find the following limits.

a) (2 points) $\lim\limits_{x\to 2} \frac{3x^2-5x-2}{x^2+7x-18}$

$$= \lim_{x\to 2} \frac{(3x+1)\cancel{(x-2)}}{\cancel{(x-2)}(x+9)}$$

$$= \lim_{x\to 2} \frac{3x+1}{x+9} = \frac{3(2)+1}{2+9}$$

Answer: $\frac{7}{11}$

b) (2 points) $\lim\limits_{x\to 2^-} \frac{x^2-2x-6}{x-2}$

$$\frac{\text{top} \to -6}{\text{bot} \to 0,\ \text{neg}}$$

Answer: $\infty$

c) (2 points) $\lim\limits_{x\to \infty} \frac{\sqrt{4x^2+3x-1}}{3x+2}$

$$= \lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+3x-1}}{x}}{\frac{3x+2}{x}}$$

$$\frac{\sqrt{4x^2+3x-1}}{\sqrt{x^2}} = \sqrt{\frac{4x^2+3x-1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{4+\frac{3}{x}-\frac{1}{x^2}}}{3+\frac{2}{x}}$$

$$= \frac{\sqrt{4}}{3}$$

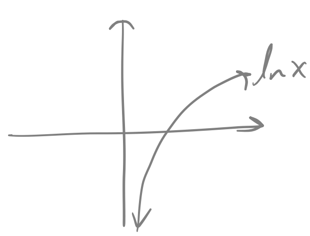
Answer: $\frac{2}{3}$

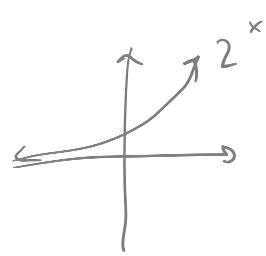
d) (2 points) $\lim\limits_{x\to 1^+} 2^{\ln(x-1)}$

$x-1 \to 0^+$

$\ln(x-1) \to -\infty$

$2^{\ln(x-1)} \to 0$




Answer: $0$

a) (2 points) $\lim_{x\to-\infty}\left(\sqrt{x^2+1}+x\right)$ (Be sure to explain your reasoning for full credit here!)

$$= \lim_{x\to-\infty} \frac{(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)}{(1)(\sqrt{x^2+1}-x)}$$

Answer: $0$

$$= \lim_{x\to-\infty} \frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1}-x}$$

2. (2 points) Find the value of $c$ such that $f(x) = \begin{cases} 2+\sqrt{x} & \text{if } 0 \le x \le 1 \\ \ln cx & \text{if } x > 1 \end{cases}$ is continuous on $[0,\infty)$

$c =$ $e^3$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-}(2+\sqrt{x}) = 2+\sqrt{1} = 3$$

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} \ln cx = \ln c(1) = \ln c$$

Need $e^{\ln c} = e^3$

$c = e^3$

3. (3 points) Use the Intermediate Value Theorem to show that there is a root of $3-2x=e^x$ in the interval $(0,1)$.

$$\underbrace{e^x+2x-3}_{f(x)} = 0$$

$f(0) = e^0+2(0)-3 = -2 < 0$

$f(1) = e^1+2(1)-3 = e-1 > 0$

Since $f$ is continuous and $f(0)<0$ and $f(1)>0$, we must have an $x$ in $(0,1)$ such that $f(x)=0$ by IVT. That is there must be a root of $3-2x=e^x$ in $(0,1)$. ▨

4. (4 points) Find the derivative of $f(x) = \frac{2}{x-1}$ <u>using the limit definition</u>.

Answer: $\frac{-2}{(x-1)^2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{2}{x+h-1} - \frac{2}{x-1}\right)\cdot(x+h-1)(x-1)}{(h)\cdot(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{2(x-1) - 2(x+h-1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x} - \cancel{2} - \cancel{2x} - 2h + \cancel{2}}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{-2}{(x+h-1)(x-1)} = \frac{-2}{(x-1)(x-1)}$$

5. (3 points) Find an equation for the tangent line to $y = (5x^2 + 2)e^x$ at the point $(0, 2)$.

Answer: $y - 2 = 2x$

$(\text{or } y = 2x + 2)$

$$\frac{dy}{dx} = (5x^2+2)e^x + 10xe^x$$

$$\left.\frac{dy}{dx}\right|_{x=0} = (5(0)^2+2)e^0 + 10(0)e^0 = 2$$

6. (2 points) Find the second derivative of $y = \tan 3x$.

$y' = \sec^2(3x) \cdot 3$

Answer: $18\sec^2(3x)\tan(3x)$

$y'' = 2\sec(3x) \cdot \sec(3x)\tan(3x) \cdot 3 \cdot 3$

$= 18\sec^2(3x)\tan(3x)$

Note: Be sure to double check your work. And remember to turn in your homework and extra credit! ☺