_____ / 54 total points (both parts)

Test #1 (Part 1 – No Calculator)

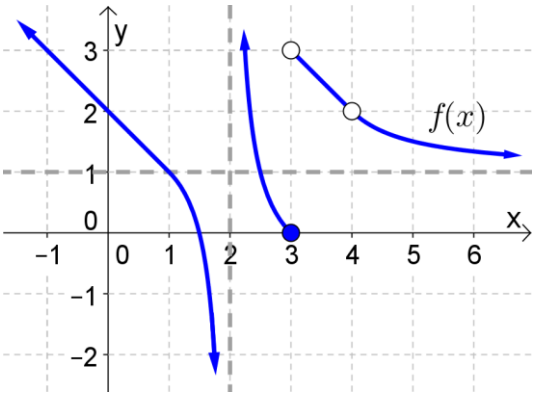
Name: _____

Math 180, Prof. Beydler

Wednesday, September 26, 2018

Directions: Show all work. No calculator, books, or notes. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. When you're finished with Part 1, please turn it in, take a bathroom break, get your calculator out, and start Part 2. Good luck!

1. (4 points) Find the following limits for the below graph of $f(x)$.



$\lim_{x\to 3^-} f(x) = 0$ $\quad \lim_{x\to 3^+} f(x) = 3$ $\quad \lim_{x\to 3} f(x)$ DNE

$\lim_{x\to 2^+} f(x) = \infty$ $\quad \lim_{x\to -1} f(x) = 3$ $\quad \lim_{x\to 4} f(x) = 2$

$\lim_{x\to -\infty} f(x) = \infty$ $\quad \lim_{x\to \infty} f(x) = 1$

2. (3 points) Using the graph of $f(x)$ from the previous question, answer the following. No need to give reasons here.

   a) Is f continuous or discontinuous at $x = 4$? continuous (discontinuous) (circle one)

   b) Is f continuous from the left at $x = 3$? (yes) no (circle one)

   c) Is f continuous from the right at $x = 2$? yes (no) (circle one)

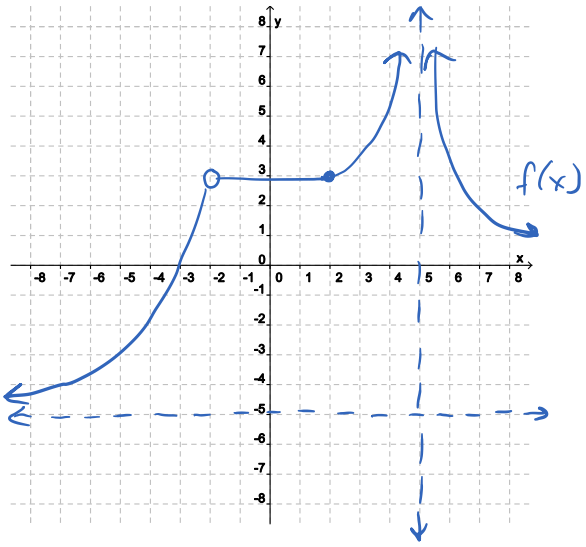
   d) Is f differentiable at $x = 4$? yes (no) (circle one)

   e) Find $f'(3.5)$. Answer: -1

   f) Find $f'(3)$. Answer: DNE

3. (2 points) Sketch the graph of an example of a function f that satisfies $\lim_{x\to -\infty} f(x) = -5$, $\lim_{x\to 5^+} f(x) = \infty$, discontinuous from the right at $x = -2$, and $f(2) = 3$. Be sure to draw any asymptotes of f.

(Answers may vary)



4. (3 points) Use the Squeeze Theorem to prove that $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$.

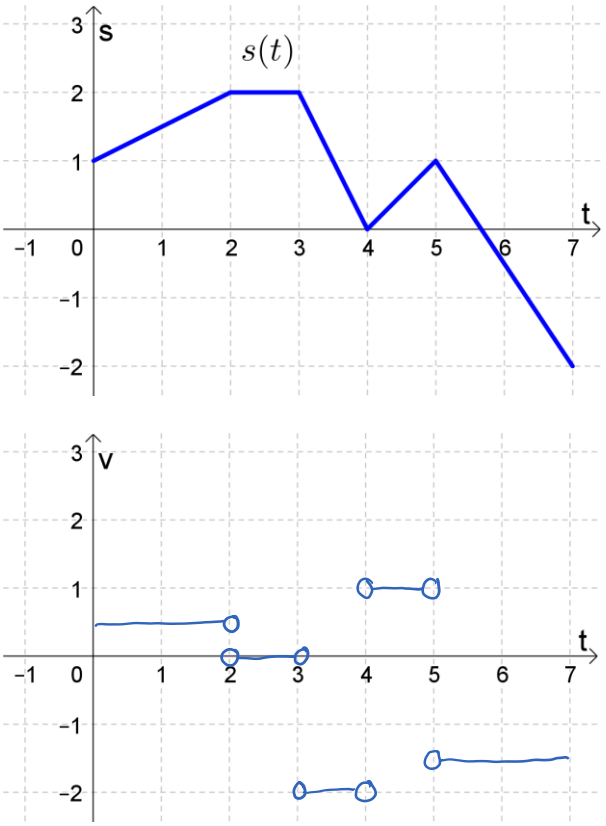
$-1 \le \cos\frac{1}{x} \le 1$

$-x^2 \le x^2 \cos\frac{1}{x} \le x^2$

Since $\lim_{x\to 0}(-x^2) = \lim_{x\to 0}(x^2) = 0$,

we have $\lim_{x\to 0} x^2 \cos\frac{1}{x} = 0$ by the Squeeze Theorem. ▨

5. (2 points) Suppose you move back and forth along a line, and that your position over time is given by the function $s(t)$ below. Graph the velocity function $v(t)$.



6. Differentiate the following functions.

a) (3 points) $f(x) = 4^x + \frac{3}{\sqrt[4]{x}} + 2e^4 + 5\ln x + \cot x + 2\cos^{-1} x$

Answer: $4^x \ln 4 - \frac{3}{4x^{5/4}} + \frac{5}{x} - \csc^2 x - \frac{2}{\sqrt{1-x^2}}$

$f(x) = 4^x + 3x^{-1/4} + 2e^4 + 5\ln x + \cot x + 2\cos^{-1} x$

$f'(x) = 4^x \ln 4 - \frac{3}{4}x^{-5/4} + 5\cdot\frac{1}{x} - \csc^2 x + 2\left(\frac{-1}{\sqrt{1-x^2}}\right)$

b) (3 points) $f(x) = \frac{x^3+\cosh x}{\sin x}$ (for this one, don't worry about simplifying your answer)

Answer: $\frac{\sin x(3x^2+\sinh x) - (x^3+\cosh x)\cos x}{\sin^2 x}$

c) (4 points) $f(x) = \frac{\csc x}{x^3 e^x}$ (for this one, don't worry about simplifying your answer)

Answer: $\frac{x^3e^x(-\csc x\cot x) - (\csc x)(x^3e^x + 3x^2e^x)}{(x^3e^x)^2}$

d) (3 points) $f(x) = \cos^2(2^x)$ (for this one, don't worry about simplifying your answer)

$= (\cos(2^x))^2$

Answer: $2(\cos(2^x))(-\sin(2^x))(2^x \ln 2)$

e) (3 points) $f(x) = \sqrt{\ln(\sec x)}$ (for this one, don't worry about simplifying your answer)

$= (\ln(\sec x))^{1/2}$

Answer: $\frac{1}{2}(\ln(\sec x))^{-1/2} \cdot \frac{1}{\sec x} \cdot \sec x \tan x$

(or $\frac{\tan x}{2\sqrt{\ln(\sec x)}}$)