_____ / 15 total points

Quiz #2 Name: _____

Math 180, Prof. Beydler Wednesday, April 22, 2020

Directions: We're working on the honor system here: no notes, books, phones, or computers during the quiz (except for using a computer to write your answers). Also, no getting help from other people. You may e-mail me to ask for clarification about any problem. **Show all work**. A **scientific calculator** is allowed. Write your answers in the indicated places, or box your answers. Good luck!

1. (4 points) Find an equation of the tangent line to $2y - 1 = e^{xy}$ at $(0, 1)$.

Answer: $y = \frac{1}{2}x + 1$

$$\frac{d}{dx}(2y) - \frac{d}{dx}(1) = \frac{d}{dx}(e^{xy})$$

$$2\frac{dy}{dx} = e^{xy} \cdot \left(x \cdot \frac{dy}{dx} + 1 \cdot y\right)$$

$$2\frac{dy}{dx} = xe^{xy}\frac{dy}{dx} + ye^{xy}$$

$$2\frac{dy}{dx} - xe^{xy}\frac{dy}{dx} = ye^{xy}$$

$$\frac{dy}{dx}(2 - xe^{xy}) = ye^{xy}$$

$$\frac{dy}{dx} = \frac{ye^{xy}}{2 - xe^{xy}}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1 \cdot e^{0 \cdot 1}}{2 - 0 \cdot e^{0 \cdot 1}} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x + 1$$

2. (4 points) Use logarithmic differentiation to find the derivative of y with respect to x.

$y = x^{\sin x}$

$\frac{dy}{dx} =$ $y\left(\frac{\sin x}{x} + \cos x \ln x\right)$

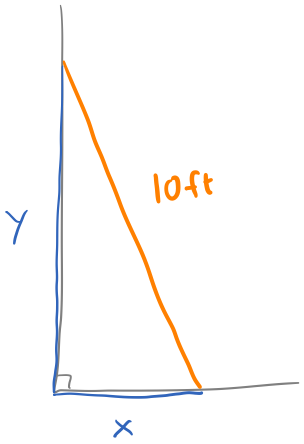
$$\ln y = \ln x^{\sin x}$$

$$= \sin x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \cos x \cdot \ln x$$

$$\frac{dy}{dx} = y\left(\frac{\sin x}{x} + \cos x \ln x\right)$$

3. (4 points) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away at 3 ft/s, how fast is the top sliding down the wall when the bottom is 2 ft from the wall? Be sure to write the units for your answer.



Answer: $0.612\ ft/s$

$$x^2+y^2=10^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=\frac{-x\frac{dx}{dt}}{y}$$

When $x=2$:

$$2^2+y^2=10^2$$

$$y=\sqrt{96}$$

$$=\frac{-2(3)}{\sqrt{96}}$$

$$\approx -0.612\ ft/s$$

4. (3 points) Prove that the derivative of $y=\sec x$ is $\frac{dy}{dx}=\sec x\tan x$ by using the derivatives of $\sin x$ and/or $\cos x$.

$$y=\sec x=\frac{1}{\cos x}$$

$$\frac{dy}{dx}=\frac{(\cos x)(0)-(1)(-\sin x)}{(\cos x)^2}$$

$$=\frac{\sin x}{\cos^2 x}$$

$$=\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}=\sec x\tan x$$ ▨