Derivatives

Basic Derivatives

Use the Memory Practice Log to cover up the right side of the table.

Note: c is any constant. a is any positive constant.

Definition of the derivative	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	It's the limit of the difference quotient.
$\frac{d}{dx}(c) =$	0	The slope of the horizontal line $y = c$ is zero.
$\frac{d}{dx}(cx) =$	c	The slope of the line $y = cx$ is c.
$\frac{d}{dx}(x^n) =$	nx^{n-1}	Power multiplies in front, and subtract one from exponent.
$\frac{d}{dx}(\sin x) =$	$\cos x$	
$\frac{d}{dx}(\cos x) =$	$-\sin x$	The derivative of trig functions starting with "c" are negative.
$\frac{d}{dx}(\tan x) =$	$\sec^2 x$	P**sst**! mnemonic. $\mathbf{s}\text{ec}\, x \to \mathbf{s}\text{ec}\, x \leftarrow \mathbf{t}\text{an}\, x$.
$\frac{d}{dx}(\sec x) =$	$\sec x \tan x$	P**sst**! mnemonic. $\mathbf{s}\text{ec}\, x \to \mathbf{s}\text{ec}\, x \leftarrow \mathbf{t}\text{an}\, x$.
$\frac{d}{dx}(\cot x) =$	$-\csc^2 x$	P**sst**!, but replace sec with csc, tan with cot, switch middle sign. $\csc x \to -\csc x \leftarrow \cot x$.
$\frac{d}{dx}(\csc x) =$	$-\csc x \cot x$	P**sst**!, but replace sec with csc, tan with cot, switch middle sign. $\csc x \to -\csc x \leftarrow \cot x$
$\frac{d}{dx}(\sin^{-1} x) =$	$\frac{1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}(\cos^{-1} x) =$	$\frac{-1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}(\tan^{-1} x) =$	$\frac{1}{1+x^2}$	The "t" in $\tan^{-1} x$ can remind you of the "+" in $1 + x^2$.
$\frac{d}{dx}(e^x) =$	e^x	
$\frac{d}{dx}(a^x) =$	$a^x \ln a$	Exponential derivative has $\ln a$ on top.
$\frac{d}{dx}(\ln x) =$	$\frac{1}{x} \quad (x > 0)$	
$\frac{d}{dx}(\log_a x) =$	$\frac{1}{x \ln a}$	Log derivative has $\ln a$ on bottom.

Hyperbolic Functions

Use the Memory Practice Log to cover up the right side of the table.

$\frac{d}{dx}(\sinh x) =$	$\cosh x$	Just like trig functions.
$\frac{d}{dx}(\cosh x) =$	$\sinh x$	Just like trig functions, but positive.
$\frac{d}{dx}(\tanh x) =$	$\operatorname{sech}^2 x$	Just like trig functions.
$\sinh x =$	$\frac{e^x - e^{-x}}{2}$	
$\cosh x =$	$\frac{e^x + e^{-x}}{2}$	Same as $\sinh x$, but with plus in middle.
$\tanh x =$	$\frac{\sinh x}{\cosh x}$	Just like trig functions.
$\operatorname{csch} x =$	$\frac{1}{\sinh x}$	Just like trig functions.
$\operatorname{sech} x =$	$\frac{1}{\cosh x}$	Just like trig functions.
$\coth x =$	$\frac{\cosh x}{\sinh x}$	Just like trig functions.
$\cosh^2 x - \sinh^2 x =$	1	Similar to $\sin^2 x + \cos^2 x = 1$, but with minus in middle and $\cosh^2 x$ written first.

Derivative Rules

Use the Memory Practice Log to cover up the right side of the table.

$(cf)' =$	cf'	**Constant Multiple Rule:** Constants can be pulled out of derivatives.
$(f \pm g)' =$	$f' \pm g'$	**Sum/Difference Rules:** Derivatives distribute across addition and subtraction.
$(fg)' =$	$fg' + f'g$	**Product Rule:** First times derivative of second plus second time derivative of first.
$\left(\frac{f}{g}\right)' =$	$\frac{gf' - fg'}{g^2}$	**Quotient Rule:** Low d high minus high d low over low squared. The Quotient Rule song! 😊
Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ OR $(f \circ g)'(x) = \underbrace{f'(g(x))}_{\text{derivative of outer}} \cdot \underbrace{g'(x)}_{\text{derivative of inner}}$	Notice that if you could cancel the du's, then you'd be back to $\frac{dy}{dx}$. Think: "Derivative of outer leaving inside alone, times derivative of inner"