Limits

Basic Limits

Use the Memory Practice Log to cover up the right side of the table.

Thinking about the graphs of each function can help a lot here.

Note: c is any constant.

$\lim_{x\to 0^+} \frac{1}{x} =$	∞
$\lim_{x\to 0^-} \frac{1}{x} =$	$-\infty$
$\lim_{x\to 0} \frac{1}{x} =$	DNE
$\lim_{x\to \infty} \frac{1}{x} =$	0
$\lim_{x\to -\infty} \frac{1}{x} =$	0
$\lim_{x\to \infty} e^x =$	∞
$\lim_{x\to -\infty} e^x =$	0
$\lim_{x\to 0^+} \ln x =$	$-\infty$
$\lim_{x\to \infty} \ln x =$	∞
$\lim_{x\to \infty} \tan^{-1} x =$	$\frac{\pi}{2}$
$\lim_{x\to -\infty} \tan^{-1} x =$	$-\frac{\pi}{2}$
$\lim_{x\to \infty} c =$	c
$\lim_{x\to -\infty} c =$	c

Limit Strategies

Use the Memory Practice Log to cover up the right side of the table.

For the following, suppose you have $\lim_{x\to c} \frac{f(x)}{g(x)}$ or $\lim_{x\to \infty} \frac{f(x)}{g(x)}$ or $\lim_{x\to -\infty} \frac{f(x)}{g(x)}$ where c is any constant.

If $f(x) \to$ nonzero # and $g(x) \to 0$, what are the possible answers?	1. $-\infty$ 2. ∞ 3. DNE (if $\lim_{x\to c^-} \frac{f(x)}{g(x)} \neq \lim_{x\to c^+} \frac{f(x)}{g(x)}$)
If $f(x) \to 0$ and $g(x) \to 0$, what are some strategies can you use?	• **Factoring:** Factor top and bottom, then cancel factors if possible. • **Conjugates:** If you see a radical (like $\sqrt{x}$), try multiplying top and bottom by conjugate, then cancel factors if possible.
If $f(x) \to \pm\infty$ and $g(x) \to \pm\infty$, what are some strategies can you use?	• **Divide:** Divide the top and bottom by the leading term of the bottom. (Note: This forces the bottom to approach a nonzero constant.)

Note: Later, we'll learn a powerful method (L'Hospital's Rule) that will help with the $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$ cases.

Squeeze Theorem

Use the Memory Practice Log to cover up the right side of the table.

What are the first 2 steps in using the Squeeze Theorem to prove that $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$?	$-1 \leq \sin\frac{1}{x} \leq 1$ So, $-x^2 \leq x^2 \sin\frac{1}{x} \leq x^2$	The first step puts bounds on part of our function $\left(\sin\frac{1}{x}\right)$. Then we multiply everything by x^2 to get an inequality that bounds the whole function $\left(x^2 \sin\frac{1}{x}\right)$.

Continuity

Use the Memory Practice Log to cover up the right side of the table.

Note: a and c are any constants.

What is the limit definition of continuity of $f(x)$ at $x = c$?	$\lim_{x\to c} f(x) = f(c)$	Intuitively, this means that the function is connected on both sides to the point $(c, f(c))$.
How do you show that a function $f(x)$ is discontinuous at $x = a$?	Show either: 1. $\lim_{x\to a} f(x)$ DNE, or 2. $f(a)$ undefined, or 3. $\lim_{x\to a} f(x) \neq f(a)$	Note: Remember that to check if $\lim_{x\to a} f(x)$ DNE, you might have to check $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$.