_____ / 30 total points

Test #3

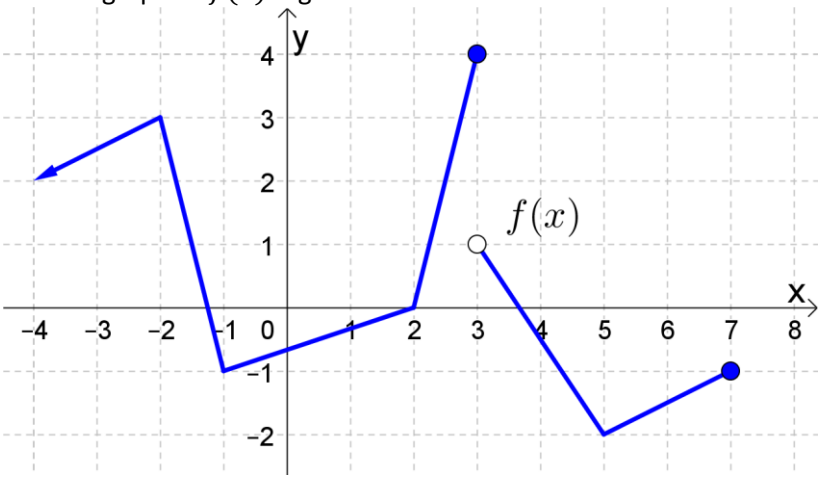
Name: _____

Math 18, Prof. Beydler

Monday, November 19, 2018

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. The graph of $f(x)$ is given below.



a) (1 point) Determine the intervals of x on which $f(x)$ is increasing, decreasing, and constant. If none, write "none".

Increasing: $(-\infty, -2], [-1, 3], [5, 7]$

Decreasing: $[-2, -1], [3, 5]$

Constant: none

b) (1 point) Find all local maxima and minima (write answers in the form $f(123) = 456$).

Local maxima: $f(-2) = 3$, $f(3) = 4$, $f(7) = -1$

Local minima: $f(-1) = -1$, $f(5) = -2$

c) (1 point) Find the absolute maximum and absolute minimum of $f(x)$ (write answers in the form $f(123) = 456$). If none, write "none".

Absolute maxima: $f(3) = 4$

Absolute minima: none

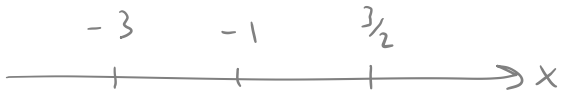
2. (3 points) Solve the following inequality.

$\frac{x+3}{2x^2-x-3} \leq 0$

$\frac{x+3}{(2x-3)(x+1)} \leq 0$ (≤ -3; $3/2$; -1)

Answer: $(-\infty, -3] \cup (-1, 3/2)$

$\frac{x+3}{(2x-3)(x+1)}$



3. Find all x-values where $f(x) = 0$ or $f(x)$ is undefined. If none, write "none".

a) (3 points) $f(x) = 2 - e^{-x}$

$f(x) = 0$: $-\ln 2$

$f(x)$ is undefined: none

$f(x) = 0$:

$2 - e^{-x} = 0$

$\ln e^{-x} = \ln 2$

$-x = \ln 2$

$x = -\ln 2$

b) (3 points) $f(x) = \ln(x^2 - 9)$

$f(x) = 0$: $\pm\sqrt{10}$

$f(x)$ is undefined: $[-3, 3]$

$f(x) = 0$:

$\ln(x^2-9) = 0$ (e, e)

$x^2 - 9 = 1$

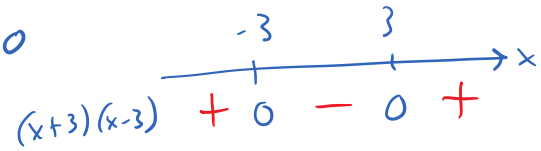
$x^2 = 10$

$x = \pm\sqrt{10}$

$f(x)$ undefined:

$x^2 - 9 \leq 0$

$(x+3)(x-3) \leq 0$ (−3, 3)



c) (3 points) $f(x) = 2x^{1/3} + 4x^{-2/3}$

$= 2x^{-2/3}(x+2)$

$= \dfrac{2(x+2)}{x^{2/3}}$

$f(x) = 0$: -2

$f(x)$ is undefined: 0

$f(x) = 0$:

$2(x+2) = 0$

$x = -2$

$f(x)$ undefined:

$x^{2/3} = 0$

$x = 0$

4. Solve the following equations.

a) (3 points) $x - 3x \ln x = 0$

$x(1 - 3\ln x) = 0$

~~$x = 0$~~ Doesn't solve original equation

$1 - 3\ln x = 0$

$-3\ln x = -1$

$e^{\ln x} = e^{1/3}$

$x = e^{1/3}$

Answer: $e^{1/3}$

b) (3 points) $5 - \frac{3}{x} - \frac{2}{x^2} = 0$

$5x^2 - 3x - 2 = 0$

$(5x+2)(x-1) = 0$

$x = \frac{-2}{5}$ $\quad x = 1$

Answer: $-\frac{2}{5}, 1$

5. (3 points) Simplify the left side of the equation and then solve.

$$\frac{x^3\frac{1}{2}(x+2)^{-1/2}-3x^2(x+2)^{1/2}}{x+2}=0$$

$\cdot\, 2(x+2)^{1/2}$

$\cdot\, 2(x+2)^{1/2}$

Answer: $0, \frac{-12}{5}$

$$\frac{x^3-6x^2(x+2)}{2(x+2)^{3/2}}=0$$

$$\frac{x^3-6x^3-12x^2}{2(x+2)^{3/2}}=0$$

$$\frac{-5x^3-12x^2}{2(x+2)^{3/2}}=0$$

$$\frac{-x^2(5x+12)}{2(x+2)^{3/2}}=0$$

$$-x^2(5x+12)=0$$

$x=0$ $\quad$ $x=\frac{-12}{5}$

6. (3 points) Find the distance from the point $(2,0)$ to the ellipse $x^2+\frac{y^2}{4}=1$ as a function of x only. Be sure to simplify your function.

Distance as a function of x: $\sqrt{8-4x-3x^2}$

$$d(x,y)=\sqrt{(x-2)^2+(y-0)^2}$$

$$d(x,y)=\sqrt{x^2-4x+4+y^2}$$

$$d(x)=\sqrt{x^2-4x+4+4-4x^2}$$

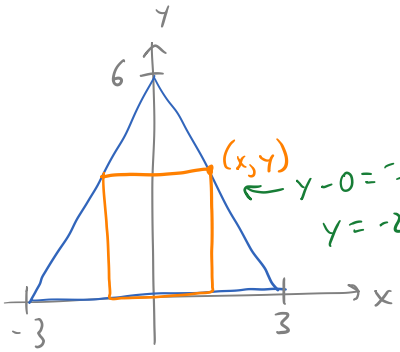
$$d(x)=\sqrt{8-4x-3x^2}$$

$$x^2+\frac{y^2}{4}=1$$

$$\frac{y^2}{4}=1-x^2$$

$$y^2=4-4x^2$$

7. (3 points) A right circular cylinder is inscribed in a cone with height 6 and base radius 3. Find a single-variable function that models the volume of the cylinder.



Volume of cylinder: $-2\pi x^3+6\pi x^2$

$y-0=\frac{-6}{3}(x-3)$

$y=-2x+6$

$$V(x,y)=\pi x^2 y$$

$y=-2x+6$

$$V(x)=\pi x^2(-2x+6)$$