

Test #2

Name: _____

Math 18, Prof. Beydler

Monday, October 15, 2018

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (2 points) Factor the following.

$$8(2x + 3)^4(x - 2) + 32(2x + 3)^3(x - 2)^2$$

Answer: 8(2x+3)³(x-2)(6x-5)

$$8(2x+3)^3(x-2)[(2x+3) + 4(x-2)]$$

2. Given $f(x) = \sin x$, $g(x) = x^2 - 3$, and $h(x) = \sqrt{1 + 2x}$, evaluate

- a) (2 points) $(h \circ f)(\pi)$

$$\begin{aligned} h(f(\pi)) &= h(\sin \pi) \\ &= h(0) \\ &= \sqrt{1 + 2(0)} \end{aligned}$$

Answer: 1

- b) (2 points) $h(f(g(x)))$

$$\begin{aligned} &= h(f(x^2 - 3)) \\ &= h(\sin(x^2 - 3)) \end{aligned}$$

Answer: $\sqrt{1 + 2\sin(x^2 - 3)}$

3. (2 points) Find $g \circ f$ for the following function. Find the domain for the composition.

$$f(x) = x^2 - 4, \quad g(x) = \frac{1}{x}$$

$g \circ f =$ $\frac{1}{x^2 - 4}$

Domain of $g \circ f$: $\{x \mid x \neq 2, -2\}$
 $(\text{or } (-\infty, -2) \cup (2, 2) \cup (2, \infty))$

Need: $x^2 - 4 \neq 0$
 $(x - 2)(x + 2) \neq 0$
 $\downarrow \quad \downarrow$
 $x \neq 2 \quad x \neq -2$

4. (2 points) Decompose the following functions. Make sure none of your functions are just x .

a) $f(g(x)) = e^{\sqrt{x}}$

$f(x) = \underline{e^x}$

$g(x) = \underline{\sqrt{x}}$

b) $f(g(h(x))) = \sin \frac{1}{(\ln x)^2}$

$f(x) = \underline{\sin x}$

$g(x) = \underline{\frac{1}{x^2}}$

$h(x) = \underline{\ln x}$

5. (2 points) Each of the following functions is a combination of two or more functions of the variable x . They are either a sum, difference, product, quotient, composition or combination of these. Describe each in words. Don't just write "product" but follow the example shown and specify the functions.

Example: $x^2 \cos(2x + 1)$ is a product of x^2 and $\cos(2x + 1)$ and a composition of $2x + 1$ inside $\cos x$

$$\frac{x^2}{\sqrt{\cos x}}$$

$\frac{x^2}{\sqrt{\cos x}}$ is a quotient of x^2 and $\sqrt{\cos x}$ and a composition of $\cos x$ inside \sqrt{x}

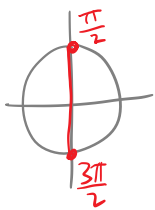
6. (3 points) Solve the following equations.

$$\cos^2 x - \sin x \cos x = 0$$

$$\cos x (\cos x - \sin x) = 0$$

$$\downarrow$$

$$\cos x = 0$$

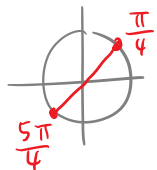


$$\downarrow$$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

$$1 = \tan x$$



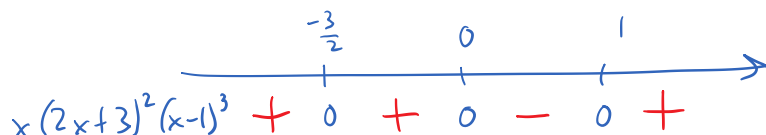
Answer: $\underline{\frac{\pi}{4} + \pi k, \frac{\pi}{2} + \pi k}$

7. (3 points) Solve the following inequality.

$$x(2x+3)^2(x-1)^3 < 0$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 0 & -\frac{3}{2} & 1 \end{array}$$

Answer: $[0, 1]$



8. (3 points) Expand the following expression using the properties of logarithms.

$$\log \sqrt{\frac{10 \sin^3 x}{3x^2 - x - 2}}$$

Answer: $\frac{1}{2} [1 + 3 \log(\sin x) - \log(3x+2) - \log(x-1)]$

$$= \frac{1}{2} \log \frac{10(\sin x)^3}{(3x+2)(x-1)}$$

$$= \frac{1}{2} \left[\underbrace{\log 10}_1 + \underbrace{\log(\sin x)^3}_{3 \log(\sin x)} - \log(3x+2) - \log(x-1) \right]$$

9. (1 point) Determine whether the following statement is true or false.

$$\ln \sqrt[4]{x+y} = \frac{1}{4} \ln x + \frac{1}{4} \ln y$$

True False (circle one)

$$(\ln \sqrt[4]{x+y} = \frac{1}{4} \ln(x+y))$$