_____ / 60 total points

Take-Home Final Exam

Math 18, Prof. Beydler

Name: _____

Wednesday, June 3, 2020

Directions: For this exam, you may use your notes. However, you may not get help from other people or other resources. Also, you may e-mail me to ask for clarification about any problem. **Show all work**. A **scientific calculator** is allowed (not a graphing calculator or program). Please write your answers in the indicated places, or box your answers. Good luck!

1. (2 points) Graph the following functions. Label intercepts and asymptotes.
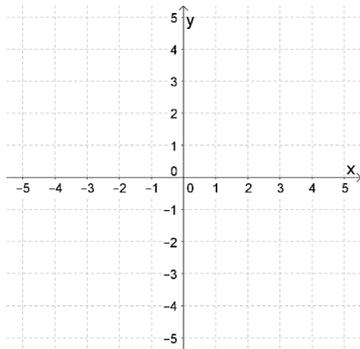
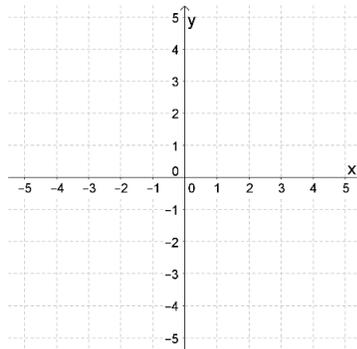a) $y = \frac{1}{x}$
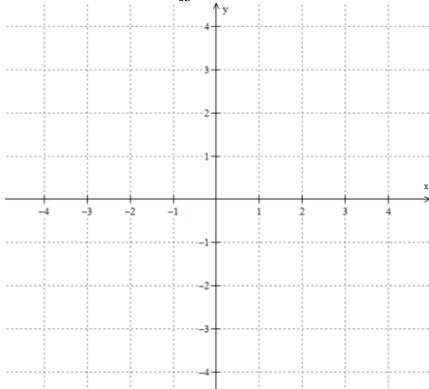
b) $y = \tan^{-1} x$
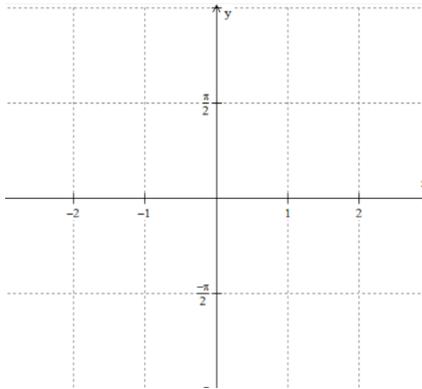
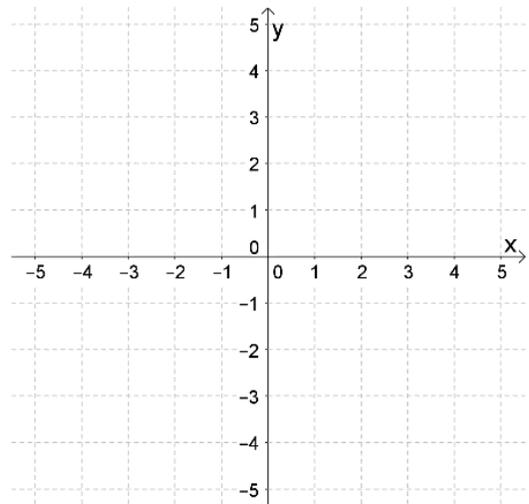2. (2 points) Graph each function.

$f(x) = -\sqrt{x-2}$

$g(x) = 1 + \ln(-x)$

3. (2 points) Graph the following piecewise function.

$f(x) = \begin{cases} 2^x + 1, & x < 1 \\ -2, & x \geq 1 \end{cases}$

4. Answer the following questions.
    a) (1 point) Rewrite $\sqrt[3]{x}$ using rational exponents. _____

    b) (1 point) Rewrite $\sqrt{x^3}$ using rational exponents. _____

5. (3 points) Find all solutions to the following equation.
    $$2 \cos 3x + \sqrt{3} = 0$$

    Answer: _____

6. (3 points) Solve the following equation.
    $$2 - 3 \ln(x + 2) = 11$$

    Answer: _____

7. (2 points) Find the equation of the line in **slope-intercept form** through the following points.
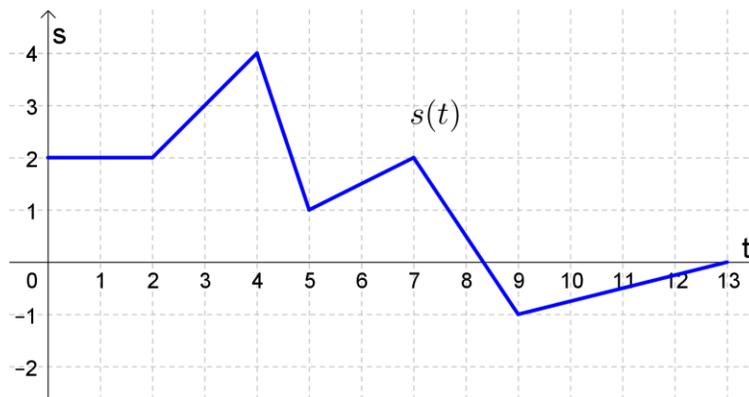 $(0, -3)$ and $(-2, 2)$

    Answer: _____

8. The position of a particle $s(t)$ (in meters) is shown to the right as a function of t (in seconds). Find the average velocity of the particle over the following time intervals. Be sure to write the units of your answers.

   a) (1 point) $t = 0$ to $t = 5$

      Answer: _____

   b) (1 point) $t = 4$ to $t = 7$

      Answer: _____

9. (3 points) Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ of $f(x) = \frac{3}{x+2}$ and simplify by canceling the factor of h.

   Answer: _____

10. (2 points) Factor the following. You will need to factor out a binomial $(a+b)^n$. Be sure to simplify each factor.

    $2(2x-1)^3(x-3) + 6(2x-1)^2(x-3)^2$

    Answer: _____

11. (2 points) Given $f(x) = x^2 - 1$, $g(x) = \frac{2}{x}$, and $h(x) = \sqrt{x+6}$ evaluate the following:

    a) $(h \circ f)(2)$

    b) $h\left(g(f(x))\right)$

12. (3 points) Solve the following equation.
$$\cos x + \sqrt{2} \sin x \cos x = 0$$

Answer: _____

13. (1 point) Determine whether each statement is true or false.
    a) $\ln(x + y) = \ln x + \ln y$ true   false   (circle one)

    b) $(\log x)^2 = 2 \log x$ true   false   (circle one)

14. (2 points) Expand the following expression using the properties of logarithms.
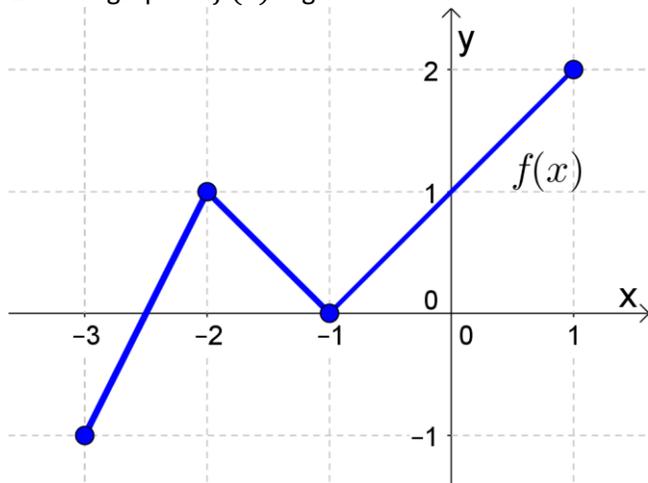$$\ln \left(\frac{(x^2 - 4x + 4) e^{\sin x}}{\sqrt[4]{x+2}} \right)$$

Answer: _____

15. (3 points) Solve the following inequality.
$$\frac{x^2 - 2x - 3}{x^3 + 2x^2} \geq 0$$

Answer: _____

16. The graph of $f(x)$ is given below.

a) (1 point) Determine the intervals of x on which $f(x)$ is increasing or decreasing (if any).

Increasing: _____

Decreasing: _____

b) (1 point) Find all local maxima and minima (write answers in the form $f(123) = 456$).

Local maxima: _____

Local minima: _____

c) (1 point) Find the absolute maximum and absolute minimum of $f(x)$, if any (write answers in the form $f(123) = 456$).

Absolute maxima: _____

Absolute minima: _____

d) (1 point) Find the domain of $f(x)$.

Domain: _____

e) (1 point) Find the range of $f(x)$.

Range: _____

17. (3 points) Solve the following equation.
$$5x^2 - x^2 \ln(x + 1) = 0$$

Answer: _____

18. Find all x-values where $f(x) = 0$ or $f(x)$ is undefined.
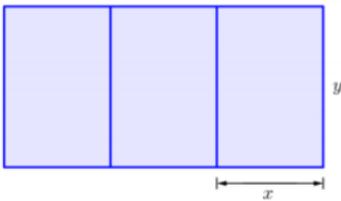   a) (3 points) $f(x) = \ln(x^2 - 4)$

$f(x) = 0:$ _____

$f(x)$ undefined: _____

   b) (3 points) $f(x) = 3x^{-2/3} + 6x^{-5/3}$

$f(x) = 0:$ _____

$f(x)$ undefined: _____

19. (3 points) A 1000-square-foot rectangular plot of land is going to be divided into three equal-sized, adjacent playgrounds (see diagram). Find the number of feet of fencing required as a function of x. (Note: there is only one fence between the playgrounds, not two.)

Answer: _____

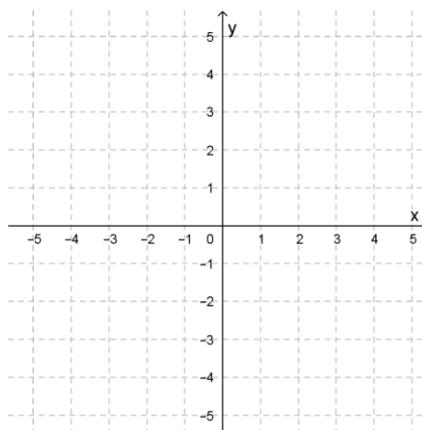20. (2 points) Evaluate $\sum_{i=0}^{2} \frac{(-1)^i (i+2)}{2^i}$

Answer: _____

21. (3 points) Simplify $\sum_{i=1}^{n}\left(\left(\frac{2i}{n}\right) - \left(\frac{2i}{n}\right)^2\right)\cdot\frac{2}{n}$

Answer: _____

22. (2 points) Graph $y = |x - 1| + 2$ from $x = -2$ to $x = 3$. Find the area between the curve and the x-axis.

Answer: _____

23. (2 points) Solve the following inequality.
$$\left|\sqrt{x - 3} - 3\right| < 2$$

Answer: _____