# Computer Lab #1

(due June 1, 2020)

For the computer labs, you'll be exploring how to use a free, open source math program called GeoGebra. GeoGebra is a Computer Algebra System (CAS), which means it can manipulate math expressions in symbolic form (with variables). Other popular CAS's include Mathematica, Maple, and Maxima. (MMM! 😊)

**Software you'll need**

1. **GeoGebra:** Visit [www.geogebra.org](http://www.geogebra.org) and under "More Apps", click on "GeoGebra Classic." GeoGebra should open up in your browser. You can also download a desktop version by clicking on "App Downloads" and then scrolling down to "GeoGebra Classic 6" and clicking "DOWNLOAD." Either one you use will be fine, though the instructions in this document will look more like the online version. However, the desktop version is more reliable and powerful.
2. **Document Editor:** For this lab, you'll also need a document editor, such as Microsoft Word, Google Docs, etc. Just pick your favorite one.

**How to insert GeoGebra images into your report**

- Use screen capture software or plugins (for example, on Windows 10 you can use the built-in snipping tool by holding down Windows key, Shift, and S), and then paste the image into your favorite document editor.
- GeoGebra Classic online version: Download the image by going to the hamburger menu (☰) on the top right, then "File," then "Download as…," then "png." Then drag the PNG file into your favorite document editor.
- GeoGebra Classic 6 desktop version: Go to "Edit," then "Graphics View to Clipboard," then paste the image into your favorite document editor (usually Ctrl-V, or Edit->Paste).

Please visit the class website to view a sample lab page to see what a lab report should look like. Remember that you'll need to include any GeoGebra commands used to answer the problems. The commands can be copied from the input bar on the left side and pasted into your report.

**Important things to know for this assignment**

- Here's how you can define a constant:

  `a=1/2` ← This lets the constant $a$ equal $\frac{1}{2}$

  `P_0=12` ← This lets the constant $P_0$ equal 12
- Here's how you can define a function:

  `f(x)=sin(ax)` ← This defines $f(x) = \sin(ax)$
- Here's how you can take the limit of a function:

  `f(x)=1/x^2` ← This defines $f(x)$ to be $\frac{1}{x^2}$

  `Limit(f, 0)` ← This finds $\lim_{x\to 0} f(x)$

`Limit(f, infinity)` ← This finds $\lim_{x \to \infty} f(x)$

`LimitAbove(1/x, 0)` ← This finds $\lim_{x \to 0^+} \frac{1}{x}$

`LimitBelow(1/x, 0)` ← This finds $\lim_{x \to 0^-} \frac{1}{x}$

- Here are a couple of ways you can adjust the $x$ scale and $y$ scale:
  - Click on  and then . Under the "Basic" tab, you can adjust the minimum and maximum $x$ and $y$ values that you see.
  - Hold shift and drag $x$-axis or $y$-axis to adjust the scale.
  - Resize your browser or desktop application window.
- If you need help, you can find GeoGebra tutorials here: https://wiki.geogebra.org/en/Tutorials

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## The Squeeze Theorem

1. Consider $f(x) = x^2 \cos\left(\frac{1}{x^3}\right)$.
   a. **Use GeoGebra** to draw the graph of $f(x)$ and adjust the $x$ and $y$ scales so the window shows approximately $-1 \leq x \leq 1$.
   b. Look at the graph from part (a) and estimate $\lim_{x \to 0} f(x)$.
   c. Use GeoGebra's "Limit" function to calculate $\lim_{x \to 0} f(x)$.
   d. Use the **Squeeze Theorem** to prove that your estimate is correct (be sure to show your reasoning). You can either write this out using pencil/pen and paper, or type up your solution and print it.
   e. Now use GeoGebra to draw the graph of $g(x) = \cos\left(\frac{1}{x^3}\right)$ with $-1 \leq x \leq 1$.
   f. Based on the graph from part (e), what is $\lim_{x \to 0} g(x)$? Why?
   g. Use GeoGebra's "Limit" function to calculate $\lim_{x \to 0} g(x)$.

## The Indeterminate Form $0^0$

We know that if $x$ is nonzero, $x^0 = 1$. We also know that if $x$ is nonzero, $0^x = 0$. What about the value of $0^0$? As mentioned in class, $0^0$ is **indeterminate** (just like $\frac{0}{0}$), but let's explore the concept a bit further by analyzing functions like $x^x$.

2. For the following parts, use $f(x) = x^x$.
   a. Use GeoGebra to draw the graph of $f$ with $-1 \leq x \leq 2$.
   b. Based on the graph from part (a), estimate $\lim_{x \to 0^+} f(x)$.
   c. Now evaluate $\lim_{x \to 0^+} f(x)$ by hand. (Note: "By hand" means to write it out on paper just like doing a homework problem, or you can type up your solution and print it.)
   d. Use GeoGebra's "`LimitAbove`" function to calculate $\lim_{x \to 0^+} f(x)$.

3. For the following parts, use $g(x) = \left(e^{-\frac{1}{x^2}}\right)^x$.
   a. Use GeoGebra to draw the graph of $g$ with $0 \leq x \leq 1$.

b. Based on the graph from part (a), estimate $\lim_{x \to 0^+} g(x)$.

c. Now evaluate $\lim_{x \to 0^+} g(x)$ by hand.

d. Use GeoGebra's "`LimitAbove`" function to calculate $\lim_{x \to 0^+} g(x)$.

4. For the following parts, use $h(x) = x^{1/\ln x}$.
   a. Use GeoGebra to draw the graph of $h$. (Note: You have to write $\ln(x)$ with parentheses in GeoGebra.)
   b. Based on the graph from part (a), estimate $\lim_{x \to 0^+} h(x)$.
   c. Now evaluate $\lim_{x \to 0^+} h(x)$ by hand.
   d. Use GeoGebra's "`LimitAbove`" function to calculate $\lim_{x \to 0^+} h(x)$.

5. Try evaluating $0^0$ into GeoGebra. What is the output?

## Modeling Population Growth

We can use the function $P(t) = P_0 e^{kt}$ to model exponential population growth over time, where $P_0$ is the population at $t = 0$, and $k$ is a relative growth constant. But what if resources (food to eat, water to drink, space to live, etc.) are limited? A given population would have a ceiling. We can use what's called a **logistic model** instead. It looks something like this:

$$P(t) = \frac{M}{1 + Ae^{-kt}} \qquad \text{where } A = \frac{M - P_0}{P_0}$$

Here, $M$ is called the **carrying capacity**, which is the maximum population possible (sometimes referred to as a population ceiling). As an interesting example of a government-imposed population ceiling see http://www.chinadaily.com.cn/china/2016-01/23/content_23215065.htm.
What about the world population ceiling? Some estimate it to be around 10 billion.

6. Suppose that the 2016 world population is 7.4 billion (use this for $P_0$), the relative growth rate $k$ is 0.015, and the carrying capacity is 10 billion.
   a. Use GeoGebra to draw the graph of the logistic model $P(t)$ with $0 \le t \le 300$.
   b. Use GeoGebra to estimate the world population in 2050 under the logistic model. (Note: Here, use the function $P(t)$ itself and not the graph of the function.)
   c. Use GeoGebra to graph $P'(t)$ with $0 \le t \le 300$. (Note: To graph the derivative, just input `P'(t)`. You might have to rescale the $y$-axis to see the graph of $P'(t)$ better.)
   d. What does the graph of $P'(t)$ represent?
   e. Based on the graph from part (d), find $\lim_{t \to \infty} P'(t)$.
   f. Use GeoGebra's "`Limit`" function to calculate $\lim_{t \to \infty} P'(t)$.