Due date: ____________ **Name:** ________________________________

Getting Ready for Derivatives (Part 4)

Notes

Recall the following properties of logarithms:

$\log_a(AB) = \log_a A + \log_a B$

$\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$

$\log_a A^C = C\log_a A$

(Note: A and B must be positive)

ex: Expand $\log_5 \frac{\sqrt[3]{x}}{25y^3}$

$$= \log_5 \frac{x^{1/3}}{25y^3}$$
$$= \log_5 x^{1/3} - \log_5(25y^3)$$
$$= \log_5 x^{1/3} - (\underbrace{\log_5 25}_{2} + \log_5 y^3)$$
$$= \boxed{\frac{1}{3}\log_5 x - 2 - 3\log_5 y}$$

ex: Expand $\ln\left(\frac{3x^2+15x+18}{e^5\cdot\sqrt[3]{x}\cdot(2x+1)^3}\right)$ $= \ln \frac{3(x+2)(x+3)}{e^5\cdot x^{1/3}\cdot(2x+1)^3}$

$3(x^2+5x+6)$

$$= \ln 3 + \ln(x+2) + \ln(x+3) - \ln e^5 - \ln x^{1/3} - \ln(2x+1)^3$$
$$= \boxed{\ln 3 + \ln(x+2) + \ln(x+3) - 5 - \frac{1}{3}\ln x - 3\ln(2x+1)}$$

1. Expand each of the following expressions using the properties of logarithms.

a) $\ln\left(\frac{(x+1)\sqrt{2x-3}}{x^2-x-2}\right)$

$$= \ln \frac{\cancel{(x+1)}(2x-3)^{1/2}}{\cancel{(x+1)}(x-2)}$$
$$= \ln \frac{(2x-3)^{1/2}}{x-2}$$
$$= \ln(2x-3)^{1/2} - \ln(x-2)$$
$$= \boxed{\frac{1}{2}\ln(2x-3) - \ln(x-2)}$$

b) $\ln\left(\sqrt{\frac{x(x-1)}{x^2+2}}\right)$

$= \ln\left(\frac{x(x-1)}{x^2+2}\right)^{1/2}$

$= \frac{1}{2}\ln\frac{x(x-1)}{x^2+2}$

$= \boxed{\frac{1}{2}\left[\ln x + \ln(x-1) - \ln(x^2+2)\right]}$

c) $\ln\left(e^5 \cdot \sqrt[3]{x} \cdot (2x^2 - 9x - 5)\right)$

$= \ln\left[e^5 \cdot x^{1/3} \cdot (2x+1)\cdot(x-5)\right]$

$= \ln e^5 + \ln x^{1/3} + \ln(2x+1) + \ln(x-5)$

$= \boxed{5 + \frac{1}{3}\ln x + \ln(2x+1) + \ln(x-5)}$

Practice at home

2. Expand each of the following expressions using the properties of logarithms.

a) $\log\left(\frac{x}{\sqrt[3]{1-x}}\right) = \log\left(\frac{x}{(1-x)^{1/3}}\right)$

$= \log x - \log(1-x)^{1/3}$

$= \boxed{\log x - \frac{1}{3}\log(1-x)}$

b) $\ln x^{\sqrt{x+1}}$

$= \boxed{\sqrt{x+1}\,\ln x}$

c) $\log_2 \frac{x^3}{3(y-1)(z+2)^2}$

$= \log_2 x^3 - \log_2 3 - \log_2(y-1) - \log_2(z+2)^2$

$= \boxed{3\log_2 x - \log_2 3 - \log_2(y-1) - 2\log_2(z+2)}$

d) $\log_3 \frac{x^3+x^2}{x^2-4}$

$= \log_3 \frac{x^2(x+1)}{(x+2)(x-2)}$

$= \log_3 x^2 + \log_3 (x+1) - \log_3 (x+2) - \log_3 (x-2)$

$= \boxed{2\log_3 x + \log_3(x+1) - \log_3(x+2) - \log_3(x-2)}$

e) $\log\sqrt{\frac{10x^2-10x-60}{(x+1)^7}}$

$10(x^2-x-6)$

$10(x+2)(x-3)$

$= \log\left[\frac{10(x+2)(x-3)}{(x+1)^7}\right]^{1/2}$

$= \frac{1}{2}\log\frac{10(x+2)(x-3)}{(x+1)^7}$

$= \frac{1}{2}\left[\log 10 + \log(x+2) + \log(x-3) - \log(x+1)^7\right]$

$= \boxed{\frac{1}{2}\left[1 + \log(x+2) + \log(x-3) - 7\log(x+1)\right]}$

f) $\ln\left(\frac{\sqrt{x}\cos^{-1}x}{2^x(x+1)}\right)$

$= \ln x^{1/2} + \ln\cos^{-1}x - \ln 2^x - \ln(x+1)$

$= \boxed{\frac{1}{2}\ln x + \ln\cos^{-1}x - x\ln 2 - \ln(x+1)}$

g) $\ln\left(\frac{\sqrt[4]{x}\cdot(2x^3+1)^2}{e^{\tan^{-1}x}}\right)$

$= \ln x^{1/4} + \ln(2x^3+1)^2 - \ln e^{\tan^{-1}x}$

$= \boxed{\frac{1}{4}\ln x + 2\ln(2x^3+1) - \tan^{-1}x}$

3. Determine whether each statement is true or false.

a) $(\ln x)^3 = 3\ln x$

$\boxed{\text{False}}$ (but $\ln x^3 = 3\ln x$ is true)

b) $\log x^2 = 2\log x$

$\boxed{\text{True}}$

c) $\ln\sqrt{x} = \frac{1}{2}\ln x$

$\boxed{\text{True}}$ (since $\ln\sqrt{x} = \ln x^{1/2} = \frac{1}{2}\ln x$)

d) $\ln\frac{x}{yz} = \ln x - \ln y + \ln z$

$\boxed{\text{False}}$ ($\ln\frac{x}{yz} = \ln x - \ln y - \ln z$)

e) $\sqrt[4]{\log x} = \frac{1}{4}\log x$

$\boxed{\text{False}}$ (but $\log\sqrt[4]{x} = \frac{1}{4}\log x$ is true)