Math 180 - Final Exam Review Exercise Answers

VISUAL CONCEPTS

1.

$\lim_{x\to 0^-} f(x) = -2$ $\lim_{x\to 0^+} f(x) = -1$ $\lim_{x\to 0} f(x)\ DNE$

$\lim_{x\to 1^-} f(x) = \infty$ $\lim_{x\to 1^+} f(x) = -\infty$ $\lim_{x\to 1} f(x)\ DNE$

$\lim_{x\to 3^-} f(x) = 0$ $\lim_{x\to 3^+} f(x) = 0$ $\lim_{x\to 3} f(x) = 0$

$\lim_{x\to 4^-} f(x) = 1$ $\lim_{x\to 4^+} f(x) = -2$ $\lim_{x\to 4} f(x)\ DNE$

$\lim_{x\to \infty} f(x) = \infty$ $\lim_{x\to -\infty} f(x) = -\infty$

2.

a) continuous
b) discontinuous
c) yes
d) yes
e) no
f) yes
g) no
h) no
i) 1
j) 0
k) DNE
l) -2
m) 2
n) $\frac{9}{2}$
o) 2
p) 0

LIMITS AND CONTINUITY

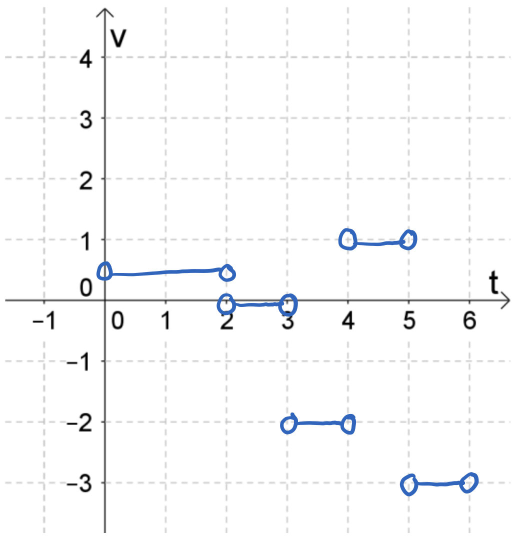
3.

a) $\frac{7}{5}$
b) $-\infty$
c) DNE
d) ∞
e) $\frac{1}{\sqrt{e}}$
f) ∞ (Hint: Divide top and bottom by x^2.)
g) 0 (Hint: Divide top and bottom by x^3. You can rewrite x^3 as $-\sqrt{x^6}$, since $x \to -\infty$.)
h) 0 (Hint: $-1 \le \cos\left(\frac{1}{x^2}\right) \le 1 \;\Rightarrow\; e^{-1} \le e^{\cos\left(\frac{1}{x^2}\right)} \le e^1 \;\Rightarrow\; x^4e^{-1} \le x^4e^{\cos\left(\frac{1}{x^2}\right)} \le x^4e^1$ …)
i) $\frac{7}{3}$ (Hint: Use L'Hospital once.)
j) $-\frac{9}{50}$ (Hint: Use L'Hospital twice.)
k) ∞ (Hint: Use L'Hospital twice.)
l) $\frac{1}{2}$ (Hint: Combine the fractions and then use L'Hospital twice.)
m) 0 (Hint: Rewrite as $\lim_{x\to 0^+} \frac{\ln x}{x^{-2}}$ then use L'Hospital.)
n) 1 (Hint: First find $\lim_{x\to 0^+} \ln(\csc x)^{\sin x}$.)
o) 1 (Hint: First find $\lim_{x\to 0^+} \ln(\cos x)^{1/x}$.)

4. $f(1) = -\frac{3}{5}$
5. $\delta = \frac{7}{4}$

DERIVATIVES

6.



7.

a) $\frac{1}{\sqrt{2x+3}}$
b) $-\frac{3}{(3x-1)^2}$

8.

a) $-\frac{3}{x^2} - \frac{5}{x} - \frac{2}{3x^{\frac{5}{3}}} + \frac{1}{\sqrt{x}} + \frac{30}{x^4}$
b) $\frac{\cos x}{2(1+\sin x)\sqrt{\sin x}}$
c) $\sec\left(\frac{e^{2x}}{x^2+3}\right)\tan\left(\frac{e^{2x}}{x^2+3}\right) \cdot \frac{2e^{2x}(x^2-x+3)}{(x^2+3)^2}$
d) $e^{x\cot x} \cdot (\cot x - x\csc^2 x)$
e) $1 + \frac{5}{x^2} + \frac{2}{x^3} - x^3 - 4x^3 \ln 3x$
f) $-5e^{-2x+3}\cos^4 x \sin x - 2e^{-2x+3}\cos^5 x$
g) $\frac{\cos x \cdot e^{4\sqrt{x}}}{x \cdot \ln x \cdot \sqrt[3]{x^2+2}} \cdot \left(-\tan x + \frac{2}{\sqrt{x}} - \frac{1}{x} - \frac{1}{x\ln x} - \frac{2x}{3(x^2+2)}\right)$ (Hint: Use logarithmic differentiation.)
h) $(\ln x)^{\tan x} \cdot \left(\frac{\tan x}{x\ln x} + \sec^2 x \cdot \ln(\ln x)\right)$ (Hint: Use logarithmic differentiation.)
i) $\frac{\coth\sqrt{x}}{2\sqrt{x}}$
j) $2\cosh 3x \operatorname{sech}^2 2x + 3\tanh 2x \sinh 3x$
k) $x\sec x + \tanh^{-1}(\sin x)$ (the first term is $\frac{x\cos x}{1-\sin^2 x}$ but that simplifies to $x\sec x$)

9.

a) $y - e = 2(x - e)$ (or $y = 2x - e$)
b) $y - \frac{\pi}{4} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right)$ (or $y = \sqrt{2}x + \frac{\pi}{4} - 1$)

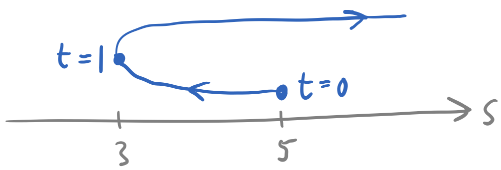
10.

a) $y = 1$

b) $y = x - 1$

11.

a) 9 m/s
b) $t = 1$ sec
c) $(1, \infty)$
d)



e) 22 m
f) 30 m/s^2

12. 40π mm^3/mm

13. 4 g/in

14. The top of the ladder is moving down the wall at a rate of $\frac{8}{3}$ m/s.

15. $\frac{32}{9\pi}$ in/min

16. Absolute max: $\frac{1}{2}$; Absolute min: $-\frac{1}{2}$

17. Absolute max: e; Absolute min: $\frac{1}{e}$

18. $f'(x) = \cos^2 x - \sin^2 x$ (Note: this is the same as $\cos 2x$). Verify hypotheses: 1. f is continuous on $\left[\frac{\pi}{3}, \frac{4\pi}{3}\right]$ since it's a product of functions that are continuous everywhere, 2. f is differentiable on $\left(\frac{\pi}{3}, \frac{4\pi}{3}\right)$ since it's the product of two functions that are differentiable everywhere, and 3. $f\left(\frac{\pi}{3}\right) = f\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{4}$. You'll have to show the work to find c, and you should get $c = \frac{3\pi}{4}$ and $c = \frac{5\pi}{4}$.

19. Since $f(x) = x^3 + \frac{4}{x^2} + 7$ is a rational function, it's continuous and differentiable on its domain $(-\infty, 0) \cup (0, \infty)$. Since $f(-3) = -\frac{176}{9} < 0$ and $f(-1) = 10 > 0$,by IVT there must be some x in $(-3, -1)$ such that $f(x) = 0$ (i.e. f has at least one zero). Now suppose there are zeros a and b in $(-\infty, 0)$, so that $f(a) = f(b) = 0$. By Rolle's Theorem, there is a c in (a, b) at which $f'(c) = 0$. But $f'(x) = 3x^2 - \frac{8}{x^3} > 0$ for all x in $(-\infty, 0)$. This is a contradiction. So, f has at most one zero. Thus, f has exactly one zero on $(-\infty, 0)$, so $x^3 + \frac{4}{x^2} + 7 = 0$ has exactly one solution on $(-\infty, 0)$. ■

20. Verify hypotheses: f is differentiable on $(0, \infty)$ since it's the product of x (differentiable on $(-\infty, \infty)$) and $\ln x$ (differentiable on $(0, \infty)$). So, 1. f is continuous on $[1, e]$, and 2. f is differentiable on $(1, e)$. You'll have to show the work to find c, and you should get $c = e^{\frac{1}{e-1}}$.

21.

a) $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

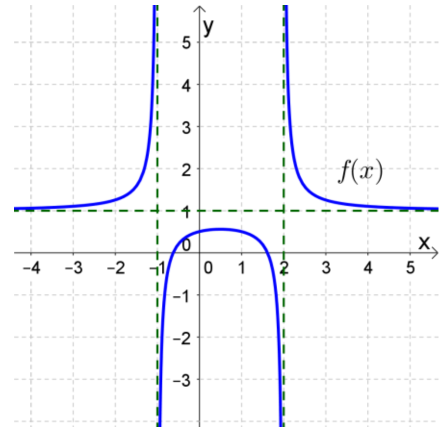
b) x-intercepts: $\left(\frac{1\pm\sqrt{5}}{2},0\right)$; y-intercept: $\left(0,\frac{1}{2}\right)$

c) Vertical asymptotes: $x=-1$ and $x=2$; Horizontal asymptote: $y=1$

d) $f'(x)=\frac{1-2x}{(x^2-x-2)^2}$ and $f''(x)=\frac{6(x^2-x+1)}{(x^2-x-2)^3}$; $f'=0$: $x=\frac{1}{2}$; f' DNE: $x=-1,2$; f'' DNE: $x=-1,2$

e)

x		-1		$\frac{1}{2}$		2	
f'	+	DNE	+	0	−	DNE	−
f''	+	DNE	−	−	−	DNE	+



f) Increasing: $(-\infty,-1)$, $\left(-1,\frac{1}{2}\right)$; Decreasing: $\left(\frac{1}{2},2\right)$, $(2,\infty)$

g) Concave up: $(-\infty,-1)$, $(2,\infty)$; Concave down: $(-1,2)$

h) Local max: $\left(\frac{1}{2},\frac{5}{9}\right)$; Local min: none; Inflection points: none

i) See graph to right.

22.

a) $(-\infty,\infty)$

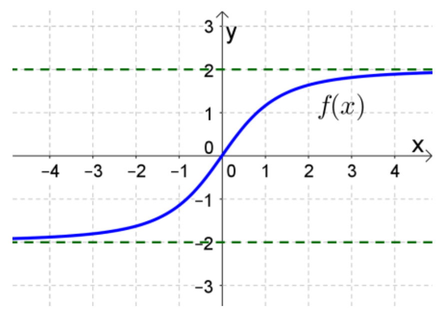
b) x-intercept: $(0,0)$; y-intercept: $(0,0)$

c) Vertical asymptotes: none; Horizontal asymptotes: $y=-2$ and $y=2$

d) $f'(x)=\frac{4}{(x^2+2)^{3/2}}$ and $f''(x)=-\frac{12x}{(x^2+2)^{5/2}}$; $f''=0$: $x=0$

e)

x		0	
f'	+	+	+
f''	+	0	−



f) Increasing: $(-\infty,\infty)$; Decreasing: nowhere

g) Concave up: $(-\infty,0)$; Concave down: $(0,\infty)$

h) Local max: none; Local min: none; Inflection points: $(0,0)$

i) See graph to right.

23.

a) $(-\infty,\infty)$

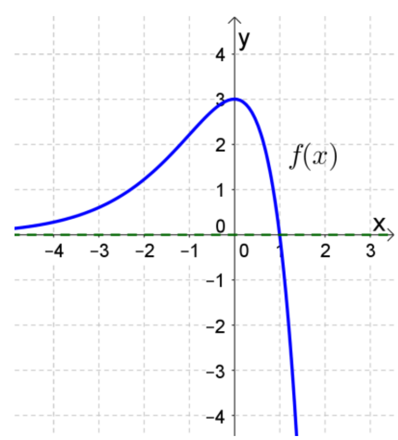
b) x-intercept: $(1,0)$; y-intercept: $(0,3)$

c) Vertical asymptotes: none; Horizontal asymptote: $y=0$

d) $f'(x)=-3xe^x$ and $f''(x)=-3e^x(x+1)$; $f'=0$: $x=0$; $f''=0$: $x=-1$

e)

x		-1		0	
f'	+	+	+	0	−
f''	+	0	−	−	−



f) Increasing: $(-\infty,0)$; Decreasing: $(0,\infty)$

g) Concave up: $(-\infty,-1)$; Concave down: $(-1,\infty)$

h) Local max: $(0,3)$; Local min: none; Inflection points: $\left(-1,\frac{6}{e}\right)$

i) See graph to right.

24. $\frac{256}{5\sqrt[4]{5}}$ (Hint: If you use a coordinate system, $A(x,y) = 2xy$. Then use $y = 16 - x^4$ to get $A(x) = 2x(16 - x^4)$.)

25. -1.452626

INTEGRALS

26.
   a) $s(t) = t^2 - \tan^{-1} t + 1$
   b) $s(t) = -\sin t - 3\cos t + 3t + 3$

27. $f(x) = \frac{x^5}{10} + \frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2} - \frac{251}{60}x + 2$

28.
   a) 1.007452
   b) 1.556758
   c) 1.302645

29.
   a) 4.35 gallons
   b) 3.8 gallons

30.
   a) $\frac{95}{3}$ (Note: $\Delta x = \frac{5}{n}$ and $x_i = \frac{5i}{n}$)
   b) 10 (Note: $\Delta x = \frac{2}{n}$ and $x_i = \frac{2i}{n}$)

31.
   a) -20
   b) 6

32.
   a) $\frac{9\pi}{2} + 6$
   b) $\frac{33}{2}$

33.
   a) $-\frac{2}{3x^3} - \frac{6}{\sqrt{x}} - \frac{1}{4}\ln|x| + 4x^{5/4} + C$
   b) $3\sin\left(\frac{1}{3}x\right) - 3\csc x + \frac{1}{2}e^{-2x+1} + \frac{2^x}{\ln 2} + C$
   c) $\frac{3}{2}\tan^{-1} x + \frac{1}{5}\sin^{-1} x + \tan x + C$
   d) $2\sqrt{x} - \frac{3}{4}x^4 + C$ (Hint: Rewrite as $\int\left(\frac{\sqrt{x}}{x} - \frac{3x^4}{x}\right)dx$ and then $\int\left(x^{-\frac{1}{2}} - 3x^3\right)dx$
   e) $4\ln 2 - 1$
   f) $\frac{\sqrt{6}-2}{\sqrt{3}}$
   g) $2(e-1)$
   h) $\frac{52}{9}$ (Hint: Substitution with $u = 1 + x^3$.)
   i) $2e^{\sqrt{x+2}} + C$ (Hint: Substitution with $u = \sqrt{x+2}$.)

j) $\frac{\sin^5 \pi x}{5\pi} + C$ (Hint: Substitution with $u = \sin \pi x$.)
k) 0 (Hint: $\frac{x^2 \tan x}{3+\cos x}$ is odd.)
l) $\frac{1}{2}\left(\ln\left(3+\sqrt{2}\right) - \ln 4\right)$ (or $\ln\frac{\sqrt{3+\sqrt{2}}}{2}$) (Hint: Substitution with $u = 3 + \sec 2x$.)
m) $-\frac{1}{2}\cos(\ln x) + C$ (Hint: Substitution with $u = \ln x$.)
n) $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x + C$ (Hint: Integration by parts.)
o) $-\frac{1}{10}e^{-x}\sin 3x - \frac{3}{10}e^{-x}\cos 3x + C$ (Hint: Integration by parts.)
p) $\frac{4}{15}$ (Hint: Either substitution with $u = 1 - x$ or integration by parts.)
q) $x \tan x + \ln(\cos x) + C$ (Hint: Integration by parts, then substitution to find $\int \tan x \, dx$.)
r) $(x^2+1)\ln(x^2+1) - x^2 - 1 + C$ (Note: This can also be written just $(x^2+1)\ln(x^2+1) - x^2 + C$)
(Hint: Substitution with $u = x^2 + 1$, then integration by parts to find $\int \ln u \, du$.)

34. Displacement: 0 meters; Distance: 20 meters