

Math 180 - Final Exam Review Exercise Answers

VISUAL CONCEPTS

1.

$$\lim_{x \rightarrow 0^-} f(x) = -2$$

$$\lim_{x \rightarrow 1^-} f(x) = \infty$$

$$\lim_{x \rightarrow 3^-} f(x) = 0$$

$$\lim_{x \rightarrow 4^-} f(x) = 1$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow 0^+} f(x) = -1$$

$$\lim_{x \rightarrow 1^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = 0$$

$$\lim_{x \rightarrow 4^+} f(x) = -2$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$\lim_{x \rightarrow 0} f(x) \text{ DNE}$$

$$\lim_{x \rightarrow 1} f(x) \text{ DNE}$$

$$\lim_{x \rightarrow 3} f(x) = 0$$

$$\lim_{x \rightarrow 4} f(x) \text{ DNE}$$

2.

- a) continuous
- b) discontinuous
- c) yes
- d) yes
- e) no
- f) yes
- g) no
- h) no
- i) 1
- j) 0
- k) DNE
- l) -2
- m) 2
- n) $\frac{9}{2}$
- o) 2
- p) 0

LIMITS AND CONTINUITY

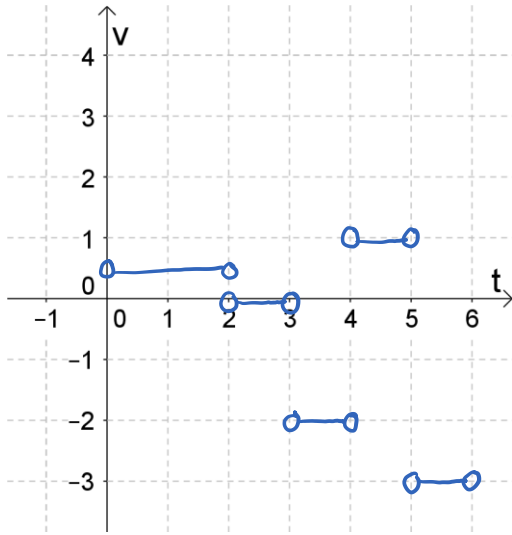
3.

- a) $\frac{7}{5}$
- b) $-\infty$
- c) DNE
- d) ∞
- e) $\frac{1}{\sqrt{e}}$
- f) ∞
- g) 0
- h) 0
- i) $\frac{7}{3}$
- j) $-\frac{9}{50}$
- k) ∞
- l) $\frac{1}{2}$
- m) 0
- n) 1
- o) 1

4. $f(1) = -\frac{3}{5}$

DERIVATIVES

5.



6.

- a) $\frac{1}{\sqrt{2x+3}}$
- b) $-\frac{3}{(3x-1)^2}$

7.

- a) $-\frac{3}{x^2} - \frac{5}{x} - \frac{2}{3x^{\frac{5}{3}}} + \frac{1}{\sqrt{x}} + \frac{30}{x^4}$
- b) $\frac{\cos x}{2(1+\sin x)\sqrt{\sin x}}$
- c) $\sec\left(\frac{e^{2x}}{x^2+3}\right) \tan\left(\frac{e^{2x}}{x^2+3}\right) \cdot \frac{2e^{2x}(x^2-x+3)}{(x^2+3)^2}$
- d) $e^{x \cot x} \cdot (\cot x - x \csc^2 x)$
- e) $1 + \frac{5}{x^2} + \frac{2}{x^3} - x^3 - 4x^3 \ln 3x$
- f) $-5e^{-2x+3} \cos^4 x \sin x - 2e^{-2x+3} \cos^5 x$
- g) $\frac{\cos x \cdot e^{4\sqrt{x}}}{x \cdot \ln x \cdot \sqrt[3]{x^2+2}} \cdot \left(-\tan x + \frac{2}{\sqrt{x}} - \frac{1}{x} - \frac{1}{x \ln x} - \frac{2x}{3(x^2+2)}\right)$
- h) $(\ln x)^{\tan x} \cdot \left(\frac{\tan x}{x \ln x} + \sec^2 x \cdot \ln(\ln x)\right)$
- i) $\frac{\coth \sqrt{x}}{2\sqrt{x}}$
- j) $2 \cosh 3x \operatorname{sech}^2 2x + 3 \tanh 2x \sinh 3x$
- k) $x \sec x + \tanh^{-1}(\sin x)$ (the first term is $\frac{x \cos x}{1-\sin^2 x}$ but that simplifies to $x \sec x$)

8.

- a) $y - e = 2(x - e)$ (or $y = 2x - e$)
- b) $y - \frac{\pi}{4} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right)$ (or $y = \sqrt{2}x + \frac{\pi}{4} - 1$)

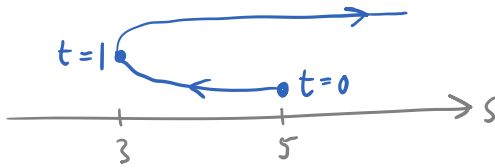
9.

- a) $y = 1$
- b) $y = x - 1$

10.

- a) 9 m/s

- b) $t = 1$ sec
 c) $(1, \infty)$
 d)



- e) 22 m
 f) 30 m/s^2

11. $40\pi \text{ mm}^3/\text{mm}$

12. 4 g/in

13. The top of the ladder is moving down the wall at a rate of $\frac{8}{3} \text{ m/s}$.

14. $\frac{147}{\sqrt{457}} \text{ mph}$ ($\approx 6.876 \text{ mph}$)

15. $\frac{32}{9\pi} \text{ in/min}$

16. Absolute max: $\frac{1}{2}$; Absolute min: $-\frac{1}{2}$

17. Absolute max: e ; Absolute min: $\frac{1}{e}$

18. $f'(x) = \cos^2 x - \sin^2 x$ (Note: this is the same as $\cos 2x$). Verify hypotheses: 1. f is continuous on $\left[\frac{\pi}{3}, \frac{4\pi}{3}\right]$ since it's a product of functions that are continuous everywhere, 2. f is differentiable on $\left(\frac{\pi}{3}, \frac{4\pi}{3}\right)$ since it's the product of two functions that are differentiable everywhere, and 3. $f\left(\frac{\pi}{3}\right) = f\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{4}$. You'll have to show the work to find c , and you should get $c = \frac{3\pi}{4}$ and $c = \frac{5\pi}{4}$.

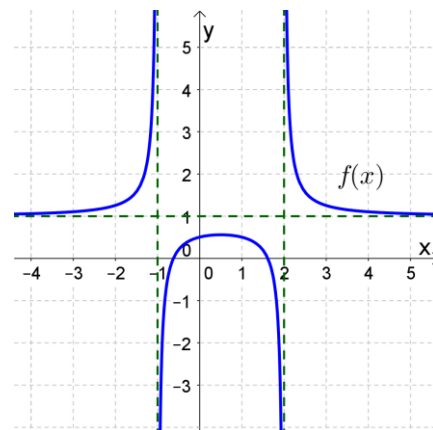
19. Since $f(x) = x^3 + \frac{4}{x^2} + 7$ is a rational function, it's continuous and differentiable on its domain $(-\infty, 0) \cup (0, \infty)$. Since $f(-3) = -\frac{176}{9} < 0$ and $f(-1) = 10 > 0$, by IVT there must be some x in $(-3, -1)$ such that $f(x) = 0$ (i.e. f has at least one zero). Now suppose there are zeros a and b in $(-\infty, 0)$, so that $f(a) = f(b) = 0$. By Rolle's Theorem, there is a c in (a, b) at which $f'(c) = 0$. But $f'(x) = 3x^2 - \frac{8}{x^3} > 0$ for all x in $(-\infty, 0)$. This is a contradiction. So, f has at most one zero. Thus, f has exactly one zero on $(-\infty, 0)$, so $x^3 + \frac{4}{x^2} + 7 = 0$ has exactly one solution on $(-\infty, 0)$. ■

20. Verify hypotheses: f is differentiable on $(0, \infty)$ since it's the product of x (differentiable on $(-\infty, \infty)$) and $\ln x$ (differentiable on $(0, \infty)$). So, 1. f is continuous on $[1, e]$, and 2. f is differentiable on $(1, e)$. You'll have to show the work to find c , and you should get $c = e^{\frac{1}{e-1}}$.

21.

- a) $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$
- b) x-intercepts: $(\frac{1 \pm \sqrt{5}}{2}, 0)$; y-intercept: $(0, \frac{1}{2})$
- c) Vertical asymptotes: $x = -1$ and $x = 2$; Horizontal asymptote: $y = 1$
- d) $f'(x) = \frac{1-2x}{(x^2-x-2)^2}$ and $f''(x) = \frac{6(x^2-x+1)}{(x^2-x-2)^3}$; $f' = 0$: $x = \frac{1}{2}$; f' DNE: $x = -1, 2$; f'' DNE: $x = -1, 2$
- e)

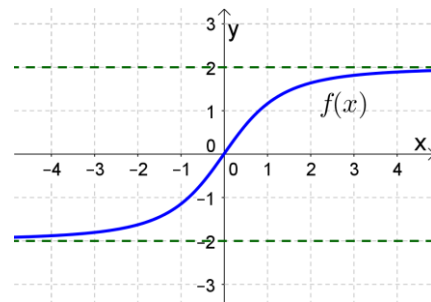
		-1		$\frac{1}{2}$		2		
f'	+	DNE	+	0	-	DNE	-	
f''	+	DNE	-	-	-	DNE	+	
- f) Increasing: $(-\infty, -1), (-1, \frac{1}{2})$; Decreasing: $(\frac{1}{2}, 2), (2, \infty)$
- g) Concave up: $(-\infty, -1), (2, \infty)$; Concave down: $(-1, 2)$
- h) Local max: $(\frac{1}{2}, \frac{5}{9})$; Local min: none; Inflection points: none
- i) See graph to right.



22.

- a) $(-\infty, \infty)$
- b) x-intercept: $(0, 0)$; y-intercept: $(0, 0)$
- c) Vertical asymptotes: none; Horizontal asymptotes: $y = -2$ and $y = 2$
- d) $f'(x) = \frac{4}{(x^2+2)^{3/2}}$ and $f''(x) = -\frac{12x}{(x^2+2)^{5/2}}$; $f'' = 0$: $x = 0$
- e)

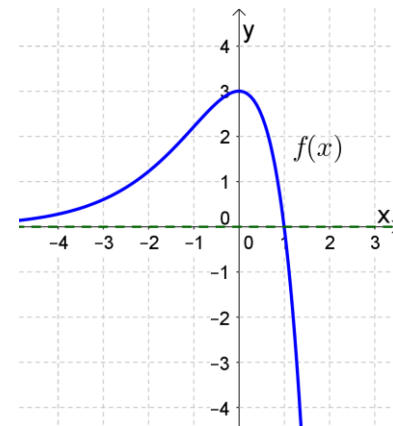
f'	+	+	+	
f''	+	0	-	
- f) Increasing: $(-\infty, \infty)$; Decreasing: nowhere
- g) Concave up: $(-\infty, 0)$; Concave down: $(0, \infty)$
- h) Local max: none; Local min: none; Inflection points: $(0, 0)$
- i) See graph to right.



23.

- a) $(-\infty, \infty)$
- b) x-intercept: $(1, 0)$; y-intercept: $(0, 3)$
- c) Vertical asymptotes: none; Horizontal asymptote: $y = 0$
- d) $f'(x) = -3xe^x$ and $f''(x) = -3e^x(x+1)$; $f' = 0$: $x = 0$; $f'' = 0$: $x = -1$
- e)

		-1		0	
f'	+	+	+	0	-
f''	+	0	-	-	-
- f) Increasing: $(-\infty, 0)$; Decreasing: $(0, \infty)$
- g) Concave up: $(-\infty, -1)$; Concave down: $(-1, \infty)$
- h) Local max: $(0, 3)$; Local min: none; Inflection points: $(-1, \frac{6}{e})$
- i) See graph to right.



24. $\frac{256}{5\sqrt[4]{5}}$

25. $\frac{4\pi r^3}{3\sqrt{3}}$

26. The wire should be cut at $\frac{20}{4+\pi}$ cm (or $\frac{5\pi}{4+\pi}$ cm).27. -1.452626 **INTEGRALS**

28.

a) $s(t) = t^2 - \tan^{-1} t + 1$

b) $s(t) = -\sin t - 3 \cos t + 3t + 3$

29. $f(x) = \frac{x^5}{10} + \frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2} - \frac{251}{60}x + 2$

30.

a) 1.007452

b) 1.556758

c) 1.302645

31.

a) 4.35 gallons

b) 3.8 gallons

32.

a) $\frac{95}{3}$

b) 10

33.

a) -20

b) 6

34.

a) $\frac{9\pi}{2} + 6$

b) $\frac{33}{2}$

35.

a) $-2x\sqrt{x^2+3}$

b) $\frac{\sin(x+1)}{e^{x+1}} - \frac{2 \sin 2x}{e^{2x}}$

36.

a) $-\frac{2}{3x^3} - \frac{6}{\sqrt{x}} - \frac{1}{4} \ln|x| + 4x^{5/4} + C$

b) $3 \sin\left(\frac{1}{3}x\right) - 3 \csc x + \frac{1}{2}e^{-2x+1} + \frac{2^x}{\ln 2} + C$

c) $\frac{3}{2} \tan^{-1} x + \frac{1}{5} \sin^{-1} x + \tan x + C$

d) $2\sqrt{x} - \frac{3}{4}x^4 + C$

e) $4 \ln 2 - 1$

- f) $\frac{\sqrt{6}-2}{\sqrt{3}}$
- g) $2(e-1)$
- h) $\frac{52}{9}$
- i) $2e^{\sqrt{x+2}} + C$
- j) $\frac{\sin^5 \pi x}{5\pi} + C$
- k) 0
- l) $\frac{1}{2}(\ln(3 + \sqrt{2}) - \ln 4)$ (or $\ln \frac{\sqrt{3+\sqrt{2}}}{2}$)
- m) $-\frac{1}{2}\cos(\ln x) + C$
- n) $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x + C$
- o) $-\frac{1}{10}e^{-x} \sin 3x - \frac{3}{10}e^{-x} \cos 3x + C$
- p) $\frac{4}{15}$
- q) $x \tan x + \ln(\cos x) + C$
- r) $(x^2 + 1) \ln(x^2 + 1) - x^2 - 1 + C$ (Note: This can also be written just $(x^2 + 1) \ln(x^2 + 1) - x^2 + C$)

37. Displacement: 0 meters; Distance: 20 meters