Math 180 – Final Exam Info and Review Exercises

Spring 2020, Prof. Beydler

Test Info

- Final Exam Date/Time: Wednesday, June 10, 2020, 7:30am-12:30pm
- Will cover almost all packets.
- You'll have from 7:30am to 12:30pm to finish the final exam.
- I'll post Final Exam as an assignment on Canvas on the test day at 7:30am.
    - If you have a printer, you can print the exam out and write your solutions directly on it.
    - If you have a tablet, please feel free to use it to write directly into the PDF.
    - If you don't have a printer or tablet, feel free to use your own paper. Please put the exam problems in order.
    - You can submit your final exam just like you do for your homework assignments. So, you'll attach the PDF under the Final Exam assignment.
- We're working on the honor system here: no notes, books, phones, or computers during the exam.

Not on the final exam:

- Packet #10 (about proving derivatives)
- Packet #11 (about linearization and differentials)

Formulas and stuff

(Note: Know all of these except for the ones with next to them, which I'll give you. This list is not meant to include everything you'll need to know on the exam.)

Definition of the derivative: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$(cf)' = cf'$

$(f \pm g)' = f' \pm g'$

$(fg)' = fg' + gf'$

$\left(\frac{f}{g}\right)' = \frac{gf'-fg'}{g^2}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{\sinh x}{\cosh x}$

$\coth x = \frac{\cosh x}{\sinh x}$

$\operatorname{sech} x = \frac{1}{\cosh x}$

$\operatorname{csch} x = \frac{1}{\sinh x}$

$\cosh^2 x - \sinh^2 x = 1$

Here are some helpful geometry formulas to know for related rates and optimization problems:

Distance/rate/time formula: $d = rt$

Pythagorean Theorem: $a^2 + b^2 = c^2$ (or $(leg)^2 + (leg)^2 = (hypotenuse)^2$)

Area of rectangle: $A = lw$

Area of circle: $A = \pi r^2$

Area of triangle: $A = \frac{1}{2}bh$

Circumference of circle: $C = 2\pi r = \pi d$

How to get perimeter of any polygon (just add the lengths of the sides).

How to get the surface area of a 3-D surface (just add the areas of the faces/sides).

Volume of a box (also called a rectangular prism): $V = lwh$

Volume of circular cylinder: $V = \pi r^2 h$

Surface area of sphere: $S = 4\pi r^2$

Volume of sphere: $V = \frac{4}{3}\pi r^3$

Volume of cone: $V = \frac{1}{3}\pi r^2 h$

Newton's method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Summation Properties

$$\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} c a_i = c \cdot \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} c = n \cdot c$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Integral Properties

$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

$\int_a^a f(x)\,dx = 0$

$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

$\int_a^b \big(f(x) \pm g(x)\big)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

$(\min f)\cdot(b-a) \le \int_a^b f(x)\,dx \le (\max f)\cdot(b-a)$

If $f(x) \ge g(x)$ on $[a,b]$, then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$.

The Fundamental Theorem of Calculus, Part 2: $\int_a^b f(x)\,dx = F(b) - F(a)$

Substitution: $\int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du$

Integration by parts: $\int u dv = uv - \int v du$

Limit Definition: $\lim\limits_{x\to x_0} f(x) = L$ if for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \;\Rightarrow |f(x) - L| < \epsilon$.

Rolle's Theorem

Suppose that

1. $f(x)$ is continuous on $[a,b]$,
2. $f(x)$ is differentiable on (a,b), and
3. $f(a) = f(b)$.

Then there is at least one number c in (a,b) at which $f'(c) = 0$.

The Mean Value Theorem

Suppose that

1. $f(x)$ is continuous on $[a,b]$, and
2. $f(x)$ is differentiable on (a,b).

Then there is at least one point c in (a,b) at which

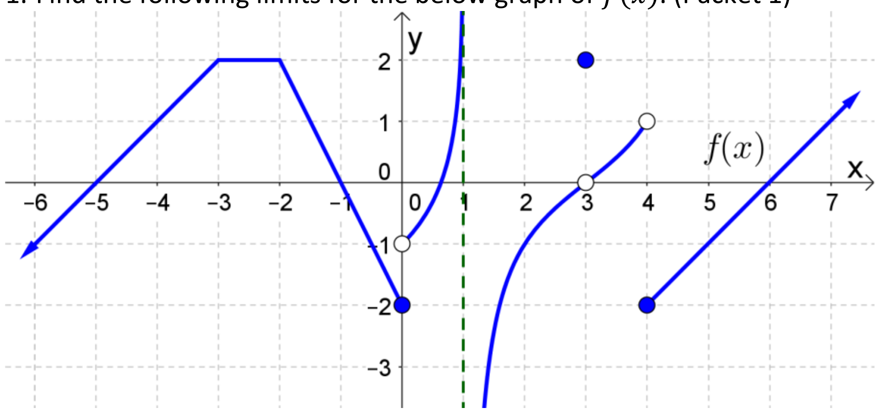
$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and submit them on Canvas on the final exam day, you can earn up to 3% extra credit towards your final exam! It is important to understand that these review exercises are <u>not guaranteed to cover all</u> of the potential problems on the final exam. Please review the notes, homework problems, previous tests, and previous test review exercises to fully prepare for the final exam.

Visual Concepts

1. Find the following limits for the below graph of $f(x)$. (Packet 1)



$\lim_{x\to 0^-} f(x)$ $\quad\lim_{x\to 0^+} f(x)$ $\quad\lim_{x\to 0} f(x)$

$\lim_{x\to 1^-} f(x)$ $\quad\lim_{x\to 1^+} f(x)$ $\quad\lim_{x\to 1} f(x)$

$\lim_{x\to 3^-} f(x)$ $\quad\lim_{x\to 3^+} f(x)$ $\quad\lim_{x\to 3} f(x)$

$\lim_{x\to 4^-} f(x)$ $\quad\lim_{x\to 4^+} f(x)$ $\quad\lim_{x\to 4} f(x)$

$\lim_{x\to \infty} f(x)$ $\quad\lim_{x\to -\infty} f(x)$

2. Using the graph of $f(x)$ from the previous question, answer the following. (Packets 3, 4, 23)

a) Is f continuous or discontinuous at $x = -2$?
b) Is f continuous or discontinuous at $x = 0$?
c) Is f continuous from the left at $x = 0$?
d) Is f continuous from the right at $x = 2$?
e) Is f continuous from the left at $x = 3$?
f) Is f differentiable at $x = 2$?
g) Is f differentiable at $x = 3$?
h) Is f differentiable at $x = 4$?
i) Find $f'(-4.1)$.
j) Find $f'(-2.5)$.
k) Find $f'(-2)$.
l) Find $f'(-1)$.
m) Find $\int_{-5}^{-3} f(x)\,dx$.
n) Find $\int_{-4}^{-1} f(x)\,dx$.
o) Find $\int_{-3}^{0} f(x)\,dx$.
p) Find $\int_{5}^{5} f(x)\,dx$.

LIMITS AND CONTINUITY

3. Find the following limits. (Packets 2 and 13)

a) $\lim\limits_{x\to 1/3} \frac{3x^2+5x-2}{6x^2+x-1}$

b) $\lim\limits_{x\to 2^+} \frac{x+1}{2-x}$

c) $\lim\limits_{x\to 2} \frac{x+1}{2-x}$

d) $\lim\limits_{x\to 2} \frac{x+1}{(2-x)^2}$

e) $\lim\limits_{x\to 0^+} e^{\frac{1}{\pi}\tan^{-1}(\ln x)}$

f) $\lim\limits_{x\to\infty} \frac{-x^3+7\sqrt{x}}{2-3x^2}$

g) $\lim\limits_{x\to -\infty} \frac{\sqrt{x^4+5x}}{x^3+2}$

h) $\lim\limits_{x\to 0} x^4 e^{\cos(1/x^2)}$ (Use the Squeeze Theorem to find/show this one.)

i) $\lim_{x \to 0} \frac{\tan 7x}{\sin 3x}$

j) $\lim_{x \to 0} \frac{1-\cos 3x}{\cos^2 5x - 1}$

k) $\lim_{x \to \infty} \frac{e^{4x}-1-4x}{x^2}$

l) $\lim_{x \to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

m) $\lim_{x \to 0^+} x^2 \ln x$

n) $\lim_{x \to 0^+} (\csc x)^{\sin x}$

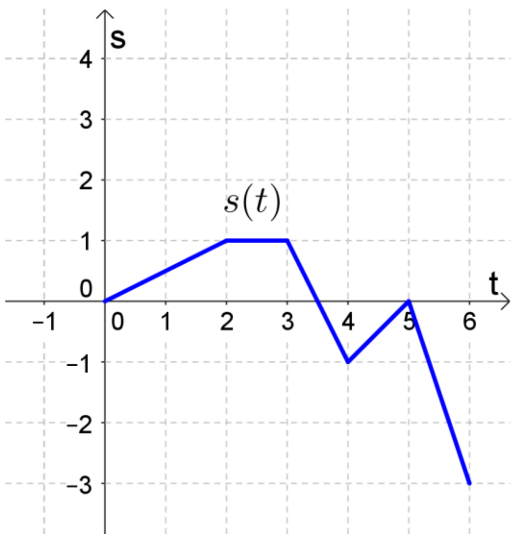
o) $\lim_{x \to 0^+} (\cos x)^{1/x}$

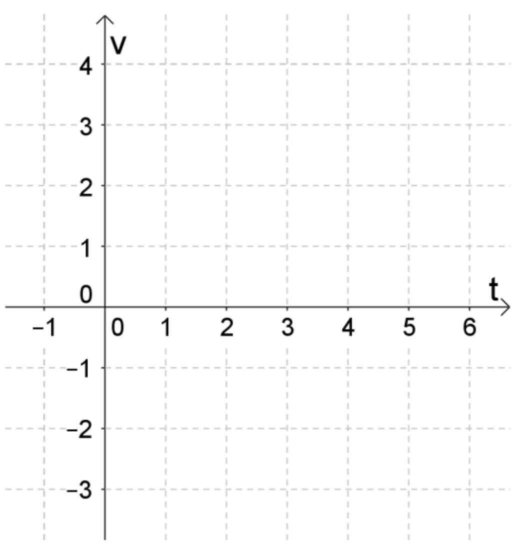
4. How would you define $f(1)$ in a way that makes $f(x) = \frac{2-x-x^2}{3x^2-x-2}$ continuous at $x = 1$? (Packet 3)

5. For $\lim_{x \to 9} \sqrt{x-5} = 2$, find the largest $\delta > 0$ that works for $\epsilon = 0.5$. (Packet 26)

DERIVATIVES

6. Suppose a bug moves back and forth along a line, and that the position of the bug over time is given by the function $s(t)$ below. Graph the velocity function $v(t)$. (Packet 4)





7. Find the derivatives of the following functions <u>using the limit definition</u>. (Packet 4)

a) $f(x) = \sqrt{2x + 3}$

b) $f(x) = \frac{1}{3x-1}$

8. Differentiate the following functions.

a) $f(x) = \frac{3}{x} - 5\ln x + \frac{1}{\sqrt[3]{x^2}} + 2\sqrt{x} - \frac{10}{x^3}$ (Packet 5)

b) $y = \tan^{-1}\left(\sqrt{\sin x}\right)$ (Packet 6)

c) $f(x) = \sec\left(\frac{e^{2x}}{x^2+3}\right)$ (Packet 6)

d) $f(x) = e^{x\cot x}$ (Packet 6)

e) $y = \frac{x^3+2x^2-5x-1}{x^2} - x^4 \ln 3x$ (Packet 6)

f) $f(x) = e^{-2x+3}\cos^5 x$ (Packet 6)

g) $y = \frac{\cos x \cdot e^{4\sqrt{x}}}{x \cdot \ln x \cdot \sqrt[3]{x^2+2}}$ (Packet 8)

h) $y = (\ln x)^{\tan x}$ (Packet 8)

i) $y = \ln\left(\frac{\sinh\sqrt{x}}{3}\right)$ (Packet 6)

j) $f(x) = \cosh 3x \tanh 2x$ (Packet 6)

k) $y = x \tanh^{-1}(\sin x)$ (Packet 6)

9. Find an equation for the tangent line to each of the following curves at each given point. (Packet 8)

   a) $y = x \ln x, \ (e, e)$

   b) $f(x) = \sin^{-1} x, \ \left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

10. Find an equation for the tangent line at the given point. (Packet 8)

    a) $\cos(xy^2) = y \ , \ (0, 1)$

b) $x^2 = \frac{x+y}{x-y}$, $(1, 0)$

11. The position of a particle is given by the equation $s(t) = t^3 - 3t + 5$ (where $t \geq 0$ is measured in seconds and s is measured in meters). (Packet 7)

a) What is the velocity after 2 seconds?

b) When is the particle at rest?

c) When is the particle moving in the positive direction?

d) Sketch a diagram to represent the motion of the particle.

e) Find the total distance traveled during the first 3 seconds.

f) Find the acceleration at time t and after 5 seconds.

12. How fast is the volume of a cylinder changing with respect to the radius when the radius is 4 mm and the height is a constant 5 mm? (Packet 7)

13. The mass of a thin rod from the left end to a point x inches to the right is $x\left(1+\sqrt{x}\right)$ grams. Find the linear density when x is 4 inches. (Packet 7)

14. A 5-meter ladder is leaning against a vertical wall. If the bottom of the ladder is pulled away from the wall at a rate of 2 meter/sec, at what rate is the top of the ladder moving down the wall when the top is 3 meters from the ground? (Packet 9)

15. Water runs into a point-down conical cup at the rate of 2 in^3/min. The cup has a height of 4 inches and a base radius of 1 inch. How fast is the water level rising when the water is 3 inches deep? (Packet 9)

16. Find the absolute maximum and minimum values of $f(x) = x\sqrt{1 - x^2}$ on the interval $[-1,1]$. (Packet 17)

17. Find the absolute maximum and minimum values of $f(x) = e^{\cos x}$ on the interval $[-\pi, 2\pi]$. (Packet 17)

18. Verify that $f(x) = \sin x \cos x$ satisfies the three hypotheses of Rolle's Theorem on $\left[\frac{\pi}{3}, \frac{4\pi}{3}\right]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem. (Packet 27)

19. Show that the equation $x^3 + \frac{4}{x^2} + 7 = 0$ has exactly one real solution on the interval $(-\infty, 0)$. (Packet 27)

20. Verify that $f(x) = x \ln x$ satisfies the hypotheses of the Mean Value Theorem on $[1, e]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem. (Packet 27)

21. Let $f(x) = \frac{1}{x^2-x-2} + 1$. (Packet 16)

a) Find the domain of f.

b) Find the x-intercept(s) and y-intercept of f (if any).

c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).

d) Find f' and f'', and determine where each are 0 and/or do not exist (DNE).

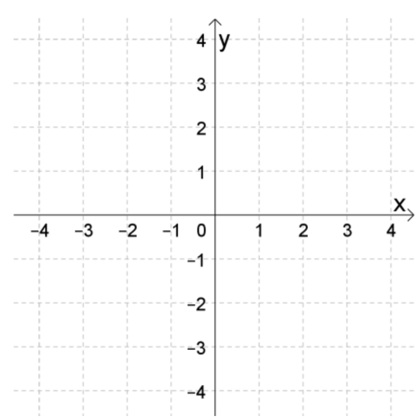
e) Do a sign analysis on f' and f''.

f) Find the intervals on which f is increasing and decreasing.

g) Find the intervals on which f is concave up and concave down.

h) Find all local maxima, local minima, and inflection points of f.

i) Sketch the graph of f.



22. Let $f(x) = \frac{2x}{\sqrt{x^2+2}}$. (Packet 16)

a) Find the domain of f.

b) Find the x-intercept(s) and y-intercept of f (if any).

c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).

d) Find f' and f'', and determine where each are 0 and/or do not exist (DNE).

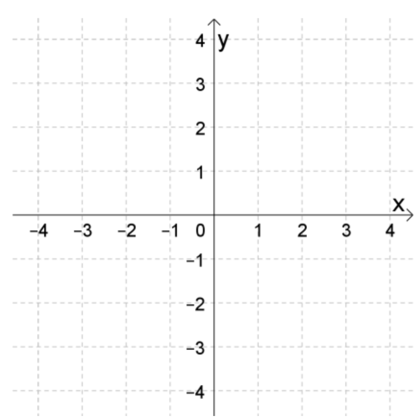
e) Do a sign analysis on f' and f''.

f) Find the intervals on which f is increasing and decreasing.

g) Find the intervals on which f is concave up and concave down.

h) Find all local maxima, local minima, and inflection points of f.

i) Sketch the graph of f.



23. Let $f(x) = 3e^x(1-x)$. (Packet 16)

a) Find the domain of f.

b) Find the x-intercept(s) and y-intercept of f (if any).

c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).

d) Find f' and f'', and determine where each are 0 and/or do not exist (DNE).

e) Do a sign analysis on f' and f''.

f) Find the intervals on which f is increasing and decreasing.

g) Find the intervals on which f is concave up and concave down.

h) Find all local maxima, local minima, and inflection points of f.

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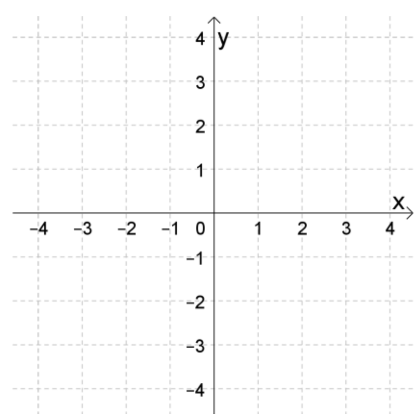

24. A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 16 - x^4$. What is the maximum area of such a rectangle? (Packet 18)

25. Use Newton's method to estimate the negative root of $x^4 + x - 3$ correct to six decimal places. Start with $x_1 = -1.5$. (Packet 12)

INTEGRALS

26. A particle is moving with the given data. Find the position of the particle. (Packet 19)

a) $v(t) = 2t - \frac{1}{1+t^2}$, $s(0) = 1$

b) $a(t) = \sin t + 3\cos t,\ s(0) = 0,\ v(0) = 2$

27. Find f if $f''(x) = 2x^3 + 3x^2 - 4x + 5$, $f(0) = 2$, and $f(1) = 0$. (Packet 24)

28. Estimate the area under the graph of $f(x) = \ln x$ between $x = 1$ and $x = 3$ using… (Packet 22)
   a) …a lower sum with four rectangles of equal width.

   b) …an upper sum with four rectangles of equal width.

   c) …midpoints with four rectangles of equal width.

29. Water is leaking out of a tank at $r(t)$ gallons per hour. Estimate the total amount of water that leaked out over 2 hours given the sample rates… (Packet 22)

t (hours)	0	0.5	1	1.5	2
$r(t)$ (gallons per hour)	2.3	2.1	2.5	1.8	1.2

   a) …using left-endpoint values.

   b) …using right-endpoint values.

30. Evaluate the following integrals using Riemann sums with right endpoints. (Packet 23)

a) $\int_0^5 (x^2 - 2)\, dx$

b) $\int_0^2 (x^3 + 3x)\, dx$

31. Suppose that $\int_{-2}^{3} f(x)\, dx = 5$ and $\int_{-2}^{-5} f(x)\, dx = 2$. Find the following. (Packet 23)

a) $\int_3^{-2} 4f(x)\, dx$

b) $\int_{-5}^{3} 2f(x)\, dx$

32. Graph the following integrands and use the area under the graph to evaluate the integral. (Packet 23)

a) $\int_{-3}^{3}\left(1+\sqrt{9-x^2}\right)dx$

b) $\int_{-5}^{0}(2+|x+3|)\,dx$

33. Evaluate the following integrals.

a) $\int\left(\frac{2}{x^4}+\frac{3}{x\sqrt{x}}-\frac{1}{4x}+5\sqrt[4]{x}\right)dx$ (Packet 19)

b) $\int\left(\cos\left(\frac{1}{3}x\right)+3\csc x\cot x-e^{-2x+1}+2^x\right)dx$ (Packet 19)

c) $\int\left(\frac{3}{2(1+x^2)}+\frac{1}{5\sqrt{1-x^2}}+\sec^2 x\right)dx$ (Packet 19)

d) $\int\frac{\sqrt{x}-3x^4}{x}\,dx$ (Packet 19)

e) $\int_{1}^{2}\left(\frac{4}{x}-\frac{2x}{3}\right)dx$ (Packet 24)

f) $\int_{\pi/4}^{\pi/3} \csc x \cot x \, dx$ (Packet 24)

g) $\int_{6}^{8} e^{\frac{1}{2}x-3} \, dx$ (Packet 24)

h) $\int_{0}^{2} x^2\sqrt{1+x^3} \, dx$ (Packet 25)

i) $\int \frac{e^{\sqrt{x+2}}}{\sqrt{x+2}} \, dx$ (Packet 20)

j) $\int \sin^4 \pi x \cos \pi x \, dx$ (Packet 20)

k) $\int_{-\pi/4}^{\pi/4} \frac{x^2 \tan x}{3+\cos x}\, dx$ (Packet 25)

l) $\int_{0}^{\pi/8} \frac{\sec 2x \tan 2x}{3+\sec 2x}\, dx$ (Packet 25)

m) $\int \frac{\sin(\ln x)}{2x}\, dx$ (Packet 20)

n) $\int x^2 \cos 2x\, dx$ (Packet 21)

o) $\int e^{-x} \sin 3x \, dx$ (Packet 21)

p) $\int_0^1 x\sqrt{1-x} \, dx$ (Packet 25)

q) $\int x \sec^2 x \, dx$ (Packet 21)

r) $\int 2x \ln(x^2 + 1)\, dx$ (Packet 21)

34. Suppose the velocity function of a particle is $v(t) = 5\cos t$ (in meters per second). Find the displacement of the particle during the time period $0 \leq t \leq 2\pi$. Then find the distance traveled by the particle during the same time period $0 \leq t \leq 2\pi$. (Packet 24)