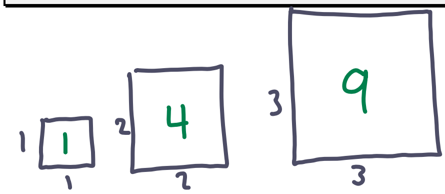


Related Rates

(covers Stewart 3.9)

Consider a square that's increasing in size over time. The area A of a square grows faster than the length x of its sides. What exactly is the relationship between these rates? We know that $A = x^2$, and if we differentiate both sides with respect to time t, we get:

$\frac{dA}{dt}, \frac{dx}{dt}$

$$\frac{d}{dt}(A) = \frac{d}{dt}(x^2) \rightarrow \frac{dA}{dt} = 2x \cdot \frac{dx}{dt}$$

How fast is the area changing when the sides have length 5 m and are changing at a rate of 1 m/s?

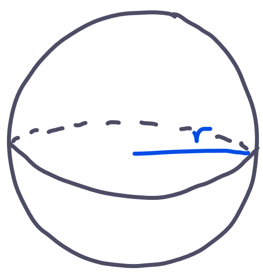
$x = 5$, $\frac{dx}{dt} = 1$

$$\left.\frac{dA}{dt}\right|_{x=5,\ \frac{dx}{dt}=1} = 2(5)(1) = \boxed{10\ m^2/s}$$

Ex 1.

Air is being pumped into a spherical balloon so the volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

$\frac{dV}{dt} = 100\ cm^3/s$



V = volume of balloon

r = radius of balloon

Goal: $\frac{dr}{dt} = ?$ (when $\frac{dV}{dt} = 100\ cm^3/s$, $r = 25\ cm$) $\quad \left(\frac{50}{2}\right)$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

↓ Chain Rule

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2} = \frac{100}{4\pi(25)^2} = \boxed{\frac{1}{25\pi}\ cm/s}$$

Ex 2.
A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away at 1 ft/s, how fast is the top sliding down the wall when the bottom is 6 ft from the wall?

Ex 3.

A conical water tank that stands point down has a base radius of 2 m and a height of 4 m. If water is pumped in at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.

Ex 4.

Car A is going west at 50 mph. Car B is going north at 60 mph. Both are going to the same intersection. At what rate are the cars approaching each other when A is 0.3 mi and B is 0.4 mi from the intersection?