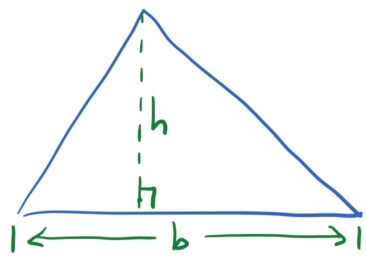
Homework for Packet #9

Due date: _____________ **Name:** ______________________________________

1. The base of a triangle is increasing at a rate of 6 mm/s and its height is decreasing by 2 mm/s. When the base is 10 mm and the height is 7 mm, is the area of the triangle increasing or decreasing? How fast is the area increasing or decreasing? Be sure to write units for your answer.



$\frac{db}{dt} = 6$, $\frac{dh}{dt} = -2$

$b = 10$, $h = 7$

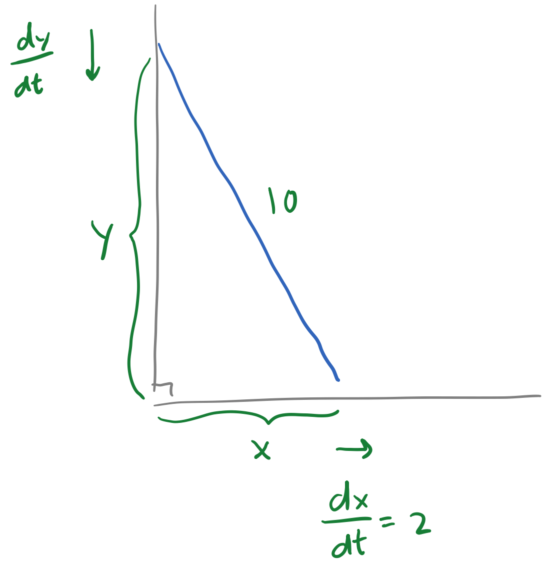
$$A = \frac{1}{2}bh$$

$$\frac{dA}{dt} = \frac{1}{2}\left(b\frac{dh}{dt} + h\frac{db}{dt}\right) = \frac{1}{2}\left(10\cdot(-2) + 7\cdot 6\right) = 11$$

The area is increasing at a rate of 11 mm^2/s.

2. The base of a triangle increases at a rate of 3 cm per minute while the height of the triangle decreases at a rate of 2 cm per minute. At what rate is the area of the triangle changing with respect to time when the height is 10 cm and the base is 15 cm? Be sure to write units for your answer.

3. A 10-meter ladder is leaning against a vertical wall. If a person pulls the base of the ladder away from the wall at a rate of 2 meter/sec, at what rate is the top of the ladder moving down the wall when the base of the ladder is 6 meters from the wall? Be sure to write units for your answer.



$$x^2 + y^2 = 10^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y}$$

When $x = 6$:

$$6^2 + y^2 = 10^2$$

$$y = 8$$

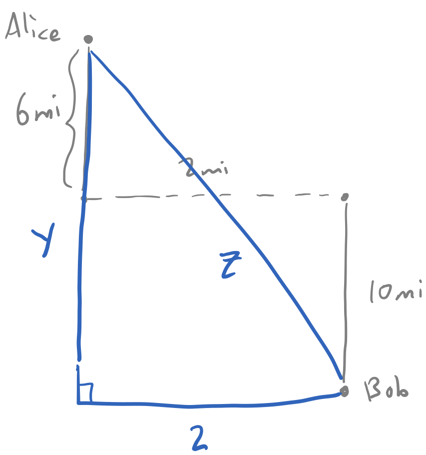
$$= \frac{-(6)(2)}{(8)}$$

$$= -3/2$$

The top of the ladder is moving down at a speed of 1.5 m/s.

4. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away at 1 ft/s, how fast is the top sliding down the wall when the bottom is 6 ft from the wall? Be sure to write units for your answer.

5. Alice is 2 miles west of Bob. At noon, Alice starts walking north at 3 mph and Bob starts jogging south at 5 mph. How fast are Alice and Bob moving apart at 2pm? Be sure to write units for your answer.



$$z^2 = y^2 + 2^2$$

$$2z\frac{dz}{dt} = 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{y\frac{dy}{dt}}{z}$$

$$= \frac{16(8)}{\sqrt{260}}$$

$$\approx 7.94$$

Find z when $y = 16$:

$$z = \sqrt{16^2 + 2^2}$$

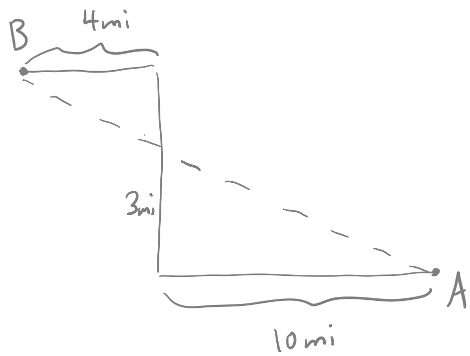
$$= \sqrt{260}$$

$\frac{dy}{dt} = 3\text{mph} + 5\text{mph} = 8\text{mph}$

$\boxed{\text{Alice and Bob are moving apart at a speed of } 7.94\text{mph.}}$

6. At 2:30pm, car A is 10 miles west of car B. Car A is moving south at 80 mph and car B is moving north at 60 mph. How fast is the distance between the cars changing at 4pm? Be sure to write units for your answer.

7. At noon, Carol is 3 miles south of Dan. Carol is running east at 5 mph and Dan is walking west at 2 mph. How fast is the distance between Carol and Dan changing at 2pm? Be sure to write units for your answer.



$$z^2 = x^2 + 3^2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt}$$

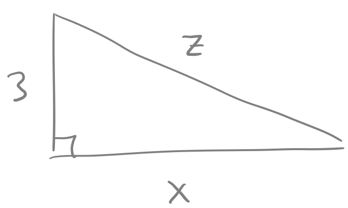
$$\frac{dz}{dt} = \frac{x\frac{dx}{dt}}{z}$$

$$z^2 = 14^2 + 3^2$$

$$z \approx 14.32$$

$$\left(\frac{98}{\sqrt{205}}\right)$$

$$\approx \frac{14(7)}{14.32}$$



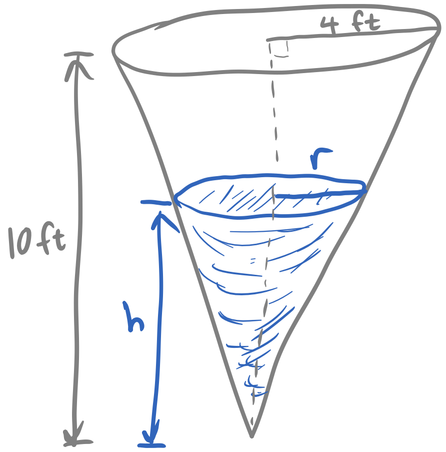
$$\approx 6.84$$

Carol and Dan are moving apart at a speed of 6.84 mph.

8. Water runs into a cylindrical cup at the rate of 3 cm^3/min. The cup has a height of 5 cm and a base radius of 1 cm. How fast is the water level rising when the water is 2 cm deep? Be sure to write units for your answer.

9. Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 4 ft. Be sure to write units for your answers below.

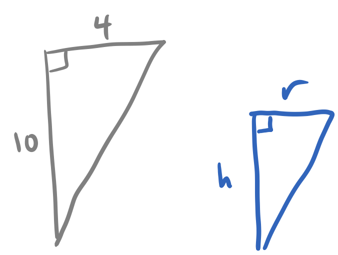
a) How fast is the water level rising when the water is 6 ft deep?



V = volume of water
h = height of water
r = radius of water surface

$$\frac{dh}{dt} = ? \quad \left(\text{when } \frac{dV}{dt} = 9 \text{ and } h = 6\right)$$

$$V = \frac{1}{3}\pi r^2 h$$



$$\frac{r}{h} = \frac{4}{10}$$

$$r = \frac{2h}{5}$$

$$V = \frac{1}{3}\pi\left(\frac{2h}{5}\right)^2 h$$

$$V = \frac{4\pi}{75} h^3$$

$$\frac{dV}{dt} = \frac{4\pi}{25} h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{25}{4\pi h^2} \cdot \frac{dV}{dt}$$

$$= \frac{25}{4\pi(6)^2} \cdot 9$$

$$= \boxed{\frac{25}{16\pi} \text{ ft/min}}$$

$$\left(\approx 0.497 \text{ ft/min}\right)$$

b) How fast is the radius of the water level changing when the water is 6 ft deep?

$$r = \frac{2}{5}h \rightarrow \frac{dr}{dt} = \frac{2}{5}\frac{dh}{dt} = \frac{2}{5}\left(\frac{25}{16\pi}\right) = \boxed{\frac{5}{8\pi} \text{ ft/min}}$$

Q: What turns everything around, but does not move? A mirror.

Optional exercises from the Stewart textbook if you'd like more practice:
3.9 (p.249) #17-33 odd