Implicit and Logarithmic Differentiation

(covers Stewart 3.5 and part of 3.6)

Recall that by the Chain Rule: $\frac{d}{dx}(x^2+1)^3 = 3(x^2+1)^2 \cdot \frac{d}{dx}(x^2+1)$

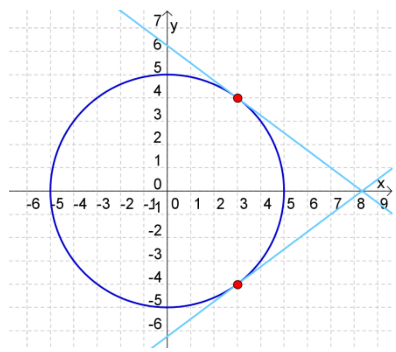
If we let $y = x^2 + 1$, then the above becomes: $\frac{d}{dx}(y^3) = 3y^2 \cdot \frac{dy}{dx}$

So, if y is a function of x, then to take the derivative of y^3 with respect to x, use the Chain Rule.

Why do we care? Because this allows us to find $\frac{dy}{dx}$ even in equations where it's difficult to solve for y explicitly—for example, the equation $x^3 + y^3 = 6xy$. The technique is called **implicit differentiation**. Let's start with a simpler example…

Ex 1.

Find $\frac{dy}{dx}$ if $x^2 + y^2 = 25$. Then find the slopes of the tangent lines at $(3, 4)$ and $(3, -4)$.

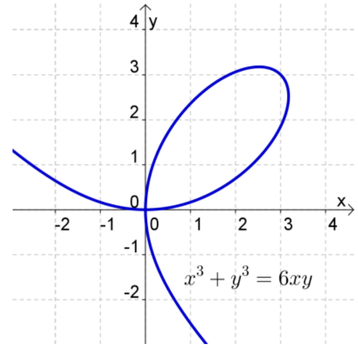


Ex 2.

Find $\frac{dy}{dx}$ if $y^2 = x^2 + \sin xy$.

Ex 3.

Find an equation of the tangent line to $x^3 + y^3 = 6xy$ at $(3,3)$.



Ex 4.

Find y'' if $2x^3 - 3y^2 = 8$.

Logarithmic Differentiation

Ex 5.

Find $\frac{dy}{dx}$ if $y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}$.