Derivative Shortcuts

(covers parts of Stewart 3.1, 3.3, 3.5, 3.6, and 3.11)

Remember these derivatives!

These three are so common you'll soon take them for granted. Here, c is a constant.

$\frac{d}{dx}(c) = 0$ $\frac{d}{dx}(cx) = c$ $\frac{d}{dx}(x^n) = nx^{n-1}$

Constant Multiple	Sum/Difference Rules	Product Rule	Quotient Rule
$(cf)' = cf'$	$(f \pm g)' = f' \pm g'$	$(fg)' = fg' + f'g$	$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Psst! Here are the trig derivatives. Notice the functions that start with "c" have negative derivatives.

$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\cot x) = -\csc^2 x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Memorize the following three inverse trig derivatives. I'll give you $\csc^{-1} x$, $\sec^{-1} x$, and $\cot^{-1} x$.

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

In Calculus, base e is exceptionally excellent!

$\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

These are called hyperbolic functions. You'll learn more about these in the homework.

$\frac{d}{dx}(\sinh x) = \cosh x$ $\frac{d}{dx}(\cosh x) = \sinh x$ $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

Don't worry about memorizing the following. I'll give them to you on quizzes and tests.

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

Ex 1.

Find the following derivatives.

$\frac{d}{dx}(-15) = \boxed{0}$

$\frac{d}{dx}(11x) = \boxed{11}$

$\frac{d}{dx}(x^3) = \boxed{3x^2}$ (exponent: $3-1$)

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = -1 \cdot x^{-2} = \boxed{\frac{-1}{x^2}}$ (exponent: $-1-1$)

$\frac{d}{dx}(x^{\pi+e}) = \boxed{(\pi+e)x^{\pi+e-1}}$

$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \boxed{\frac{1}{2\sqrt{x}}}$ (exponent: $\frac{1}{2}-1$)

Ex 2.

Find the derivative of $y = x^3 + \frac{4}{3}x^2 - 5x + 1$.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 + \frac{4}{3}x^2 - 5x + 1\right)$$

Sum/Difference Rules $(f \pm g)' = f' \pm g'$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}\left(\frac{4}{3}x^2\right) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$$

Constant Multiple Rule $(cf)' = cf'$

$$= \frac{d}{dx}(x^3) + \frac{4}{3}\frac{d}{dx}(x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$$

$$= 3x^2 + \frac{4}{3}\cdot 2x - 5 + 0$$

$$= \boxed{3x^2 + \frac{8}{3}x - 5}$$

Ex 3.

Find the derivative of $y = 4\sqrt{x} - \frac{2}{3x}$. $\leftarrow \frac{2}{3x} = \frac{2}{3}\cdot\frac{1}{x} = \frac{2}{3}x^{-1}$

$$y = 4x^{1/2} - \frac{2}{3}x^{-1}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^{1/2} - \frac{2}{3}x^{-1}\right)$$

$$= \frac{d}{dx}(4x^{1/2}) - \frac{d}{dx}\left(\frac{2}{3}x^{-1}\right)$$

$$= 4\frac{d}{dx}(x^{1/2}) - \frac{2}{3}\frac{d}{dx}(x^{-1})$$

$$= 4\cdot\frac{1}{2}x^{-1/2} - \frac{2}{3}\cdot(-1)x^{-2}$$

$$= 2x^{-1/2} + \frac{2}{3}x^{-2}$$

$$= \boxed{\frac{2}{x^{1/2}} + \frac{2}{3x^2}}$$

(OR $\frac{2}{\sqrt{x}} + \frac{2}{3x^2}$)

Ex 4.

Find the derivative of $y = 2\cos x - \sec x + 3\cot x$.

$$\frac{dy}{dx} = 2(-\sin x) - \sec x \tan x + 3(-\csc^2 x)$$
$$= \boxed{-2\sin x - \sec x \tan x - 3\csc^2 x}$$

Ex 5.

Find the derivative of $f(x) = -3\sin^{-1} x + \frac{2}{3}\tan^{-1} x$.

$$f'(x) = -3 \cdot \frac{1}{\sqrt{1-x^2}} + \frac{2}{3} \cdot \frac{1}{1+x^2}$$
$$= \boxed{\frac{-3}{\sqrt{1-x^2}} + \frac{2}{3(1+x^2)}}$$

Ex 6.

Find the derivative of $y = 4\ln x + \frac{e^x}{2} - 5^x + \frac{1}{2}\log_2 x$.

$\frac{1}{2}e^x$

$$\frac{dy}{dx} = 4 \cdot \frac{1}{x} + \frac{1}{2}e^x - 5^x \ln 5 + \frac{1}{2} \cdot \frac{1}{x \ln 5} = \boxed{\frac{4}{x} + \frac{e^x}{2} - 5^x \ln 5 + \frac{1}{2x\ln 5}}$$

Ex 7.

Find the derivative of $f(x) = -\cosh x - 7\tanh x$.

$$f'(x) = \boxed{-\sinh x - 7\operatorname{sech}^2 x}$$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + \frac{d}{dx}[f(x)]g(x)$

In other words, $(fg)' = fg' + f'g$

Ex 8.

Differentiate $y = \underbrace{\frac{1}{x}}_{f} \cdot \underbrace{(x^2 + e^x)}_{g}$.

$$\frac{dy}{dx} = \underbrace{\frac{1}{x}}_{f} \cdot \underbrace{\frac{d}{dx}(x^2+e^x)}_{g'} + \underbrace{\frac{d}{dx}\left(\frac{1}{x}\right)}_{f'} \cdot \underbrace{(x^2+e^x)}_{g}$$
$$= \frac{1}{x}(2x+e^x) + \left(\frac{-1}{x^2}\right) \cdot (x^2+e^x)$$
$$= 2 + \frac{e^x}{x} + (-1) - \frac{e^x}{x^2} = \boxed{1 + \frac{e^x}{x} - \frac{e^x}{x^2}}$$

Ex 9.

Find the derivative of $y = (x^2 + 1)(x^3 + 3)$.

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

In other words, $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Ex 10.

Differentiate $y = \frac{x^2 - 1}{x^3 + 1}$.

The Second Derivative

$f''(x) = \frac{d}{dx}[f'(x)]$ also written $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ or $y'' = (y')'$

Ex 11.
Find the second derivative of $y = 2x^4 - 5x^2 + 23x - 10$

Note: If $s(t)$ is a position function, then $s'(t)$ is the velocity function, and $s''(t)$ is the acceleration function. Acceleration measures the rate of change of velocity.

The n[th] Derivative

To get the third derivative, fourth derivative, fifth derivative, etc., just keep differentiating.
For example, here are the derivatives of $f(x) = 4x^3 - 2x^2 + 5x - 1$:

First derivative: $f'(x) = 12x^2 - 4x + 5$
Second derivative: $f''(x) = 24x - 4$
Third derivative: $f'''(x) = 24$
Fourth derivative: $f^{(4)}(x) = 0$
Fifth derivative: $f^{(5)}(x) = 0$

Ex 12.
Differentiate $y = x \sin x \cos x$.