Derivative Shortcuts

(covers parts of Stewart 3.1, 3.3, 3.5, 3.6, and 3.11)

Remember these derivatives!

These three are so common you'll soon take them for granted. Here, c is a constant.

$\frac{d}{dx}(c) = 0$ $\qquad \frac{d}{dx}(cx) = c$ $\qquad \frac{d}{dx}(x^n) = nx^{n-1}$

Constant Multiple	Sum/Difference Rules	Product Rule	Quotient Rule
$(cf)' = cf'$	$(f \pm g)' = f' \pm g'$	$(fg)' = fg' + f'g$	$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Psst! Here are the trig derivatives. Notice the functions that start with "c" have negative derivatives.

$\frac{d}{dx}(\sin x) = \cos x$ $\qquad \frac{d}{dx}(\cos x) = -\sin x$ $\qquad \frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\qquad \frac{d}{dx}(\cot x) = -\csc^2 x$ $\qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$

Memorize the following three inverse trig derivatives. I'll give you $\csc^{-1} x$, $\sec^{-1} x$, and $\cot^{-1} x$.

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

In Calculus, base e is exceptionally excellent!

$\frac{d}{dx}(e^x) = e^x$ $\qquad \frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$ $\qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

These are called hyperbolic functions. You'll learn more about these in the homework.

$\frac{d}{dx}(\sinh x) = \cosh x$ $\qquad \frac{d}{dx}(\cosh x) = \sinh x$ $\qquad \frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

Don't worry about memorizing the following. I'll give them to you on quizzes and tests.

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ $\qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ $\qquad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\qquad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ $\qquad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $\qquad \frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ $\qquad \frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

Ex 1.

Find the following derivatives.

$\frac{d}{dx}(-15) =$

$\frac{d}{dx}(11x) =$

$\frac{d}{dx}(x^3) =$

$\frac{d}{dx}\left(\frac{1}{x}\right) =$

$\frac{d}{dx}(x^{\pi+e}) =$

$\frac{d}{dx}(\sqrt{x}) =$

Ex 2.

Find the derivative of $y = x^3 + \frac{4}{3}x^2 - 5x + 1$.

Ex 3.

Find the derivative of $y = 4\sqrt{x} - \frac{2}{3x}$.

Ex 4.

Find the derivative of $y = 2\cos x - \sec x + 3\cot x$.

Ex 5.

Find the derivative of $f(x) = -3\sin^{-1} x + \frac{2}{3}\tan^{-1} x$.

Ex 6.

Find the derivative of $y = 4\ln x + \frac{e^x}{2} - 5^x + \frac{1}{2}\log_2 x$.

Ex 7.

Find the derivative of $f(x) = -\cosh x - 7\tanh x$.

Product Rule: $\boldsymbol{\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + \frac{d}{dx}[f(x)]g(x)}$

In other words, $\boldsymbol{(fg)' = fg' + f'g}$

Ex 8.

Differentiate $y = \frac{1}{x}(x^2 + e^x)$.

Ex 9.

Find the derivative of $y = (x^2 + 1)(x^3 + 3)$.

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

In other words, $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Ex 10.

Differentiate $y = \frac{x^2 - 1}{x^3 + 1}$.

The Second Derivative

$f''(x) = \frac{d}{dx}[f'(x)]$ also written $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ or $y'' = (y')'$

Ex 11.
Find the second derivative of $y = 2x^4 - 5x^2 + 23x - 10$

Note: If $s(t)$ is a position function, then $s'(t)$ is the velocity function, and $s''(t)$ is the acceleration function. Acceleration measures the rate of change of velocity.

The n^th Derivative

To get the third derivative, fourth derivative, fifth derivative, etc., just keep differentiating.
For example, here are the derivatives of $f(x) = 4x^3 - 2x^2 + 5x - 1$:

First derivative: $f'(x) = 12x^2 - 4x + 5$
Second derivative: $f''(x) = 24x - 4$
Third derivative: $f'''(x) = 24$
Fourth derivative: $f^{(4)}(x) = 0$
Fifth derivative: $f^{(5)}(x) = 0$

Ex 12.
Differentiate $y = x \sin x \cos x$.