Homework for Packet #3

1.

$x = 0$: continuous at $x = 0$ since $\lim_{x\to 0} f(x) = 0 = f(0)$
$x = 0$: continuous from the left at $x = 0$ since $\lim_{x\to 0^-} f(x) = 0 = f(0)$
$x = 0$: continuous from the right at $x = 0$ since $\lim_{x\to 0^+} f(x) = 0 = f(0)$

$x = 1$: discontinuous at $x = 1$ since $\lim_{x\to 1} f(x)$ DNE
$x = 1$: continuous from the left at $x = 1$ since $\lim_{x\to 1^-} f(x) = 1 = f(1)$
$x = 1$: discontinuous from the right at $x = 1$ since $\lim_{x\to 1^+} f(x) = 2 \neq f(1)$

$x = 3$: discontinuous at $x = 3$ since $\lim_{x\to 3} f(x) = 1 \neq f(3)$
$x = 3$: discontinuous from the left at $x = 3$ since $\lim_{x\to 3^-} f(x) = 1 \neq f(3)$
$x = 3$: discontinuous from the right at $x = 3$ since $\lim_{x\to 3^+} f(x) = 1 \neq f(3)$

2.

$x = 1$: discontinuous at $x = 1$ since $\lim_{x\to 1} f(x)$ DNE
$x = 1$: discontinuous from the left at $x = 1$ since $\lim_{x\to 1^-} f(x) = 0 \neq f(1)$
$x = 1$: continuous from the right at $x = 1$ since $\lim_{x\to 1^+} f(x) = 2 = f(1)$

$x = 2$: discontinuous at $x = 2$ since $f(2)$ is undefined
$x = 2$: discontinuous from the left at $x = 2$ since $f(2)$ is undefined
$x = 2$: discontinuous from the right at $x = 2$ since $f(2)$ is undefined

$x = 3$: discontinuous at $x = 3$ since $f(3)$ is undefined
$x = 3$: discontinuous from the left at $x = 3$ since $f(3)$ is undefined
$x = 3$: discontinuous from the right at $x = 3$ since $f(3)$ is undefined

3. See solutions.

4. $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (3 - x^2) = 3 - (-1)^2 = 2$
$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{3}{x+1} = \infty$
Since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$, $\lim_{x\to -1} f(x)$ DNE. Thus, f is discontinuous at $x = -1$.

5. $c = 0$, $c = \frac{1}{2}$

6. $c = e^3$

7. $f(-1) = -\frac{1}{4}$

8. $f(0) = 0$ (Hint: Use the Squeeze Theorem to find/show $\lim_{x\to 0} x^2 \sin\frac{1}{x}$)

9. $\lim_{x \to a} f(x) = f(a)$

10-14. See solutions.

15. True
16. False
17. True
18. False
19. False