Due date: _____ **Name:** _____

Tools for Limits (Part 2)

1. Put the following list of numbers in order from least to greatest:

$\frac{4}{10000}, \frac{4}{10000000}, \frac{4}{0.1}, \frac{4}{0.001}, 4$ $\frac{4}{10000000}$, $\frac{4}{10000}$, 4, $\frac{4}{0.1}$, $\frac{4}{0.001}$

2. Put the following list of numbers in order from least to greatest:

$\frac{10000}{4}, \frac{10000000}{4}, \frac{0.1}{4}, \frac{0.001}{4}, \frac{1}{4}$ $\frac{0.001}{4}$, $\frac{0.1}{4}$, $\frac{1}{4}$, $\frac{10000}{4}$, $\frac{10000000}{4}$

3. Answer the following questions.

a) $-1 \leq \sin x \leq 1$

b) $-1 \leq \cos x \leq 1$

c) $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

d) Problems a-c above are asking for the range of each function.

e) What is the term of $x^4 - 3x^2 + 2x + 5$ with the highest exponent? x^4

f) What is the term of $6x^2 + 2x^4 - 1$ with the highest exponent? $2x^4$

g) What is the term of $4x^2 - 3x^{3/2} + 7$ with the highest exponent? $4x^2$

h) What is the term of $2x^{1/2} + x - 5$ with the highest exponent? x

i) Rewrite $\sqrt[3]{x}$ using rational exponents. $x^{1/3}$

j) Rewrite $\sqrt[4]{x}$ using rational exponents. $x^{1/4}$

k) Rewrite $\sqrt[3]{x^2}$ using rational exponents. $x^{2/3}$

l) Rewrite $\sqrt{x^5}$ using rational exponents. $x^{5/2}$

The following type of simplification will be useful in Math 180 soon:

$\frac{x}{x^{5/2}} = x^{1-\frac{5}{2}} = x^{-3/2} = \frac{1}{x^{3/2}}$

4. Simplify $\frac{x^{2/3}}{x}$ as done above. $\frac{x^{2/3}}{x} = x^{\frac{2}{3}-1} = x^{-1/3} = \frac{1}{x^{1/3}}$

5. Simplify $\frac{\sqrt[3]{x}}{x}$ as done above. $\frac{x^{1/3}}{x} = x^{1/3-1} = x^{-2/3} = \frac{1}{x^{2/3}}$

Here's another type of simplification that will be useful in Math 180 soon:

$$\frac{\sqrt{9x^6-x}}{x^3} = \frac{\sqrt{9x^6-x}}{\sqrt{x^6}} = \sqrt{\frac{9x^6-x}{x^6}} = \sqrt{\frac{9x^6}{x^6} - \frac{x}{x^6}} = \sqrt{9 - \frac{1}{x^5}}$$

For the two examples below, assume x is positive so $x = \sqrt{x^2}$.

6. Simplify $\frac{\sqrt{4x^2+3x-1}}{x}$ as done above. $\frac{\sqrt{4x^2+3x-1}}{\sqrt{x^2}} = \sqrt{\frac{4x^2+3x-1}{x^2}} = \sqrt{\frac{4x^2}{x^2} + \frac{3x}{x^2} - \frac{1}{x^2}} = \sqrt{4 + \frac{3}{x} - \frac{1}{x^2}}$

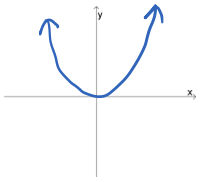
7. Simplify $\frac{\sqrt{x^3+2x^2}}{x}$ as done above. $\frac{\sqrt{x^3+2x^2}}{\sqrt{x^2}} = \sqrt{\frac{x^3+2x^2}{x^2}} = \sqrt{\frac{x^3}{x^2} + \frac{2x^2}{x^2}} = \sqrt{x+2}$

8. $x = \sqrt{x^2}$ is true when x is positive. How would this change if x were negative? $x = -\sqrt{x^2}$
(Hint: Try plugging in some negative numbers. How can you modify $x = \sqrt{x^2}$ to make it correct again?)

Practice at home

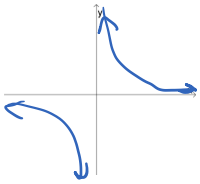
9. Write a basic example of each of the following functions, and draw a quick sketch of its graph.

a) Polynomial $f(x) =$ x^2



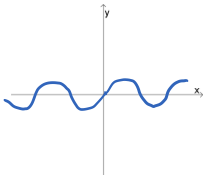
(Answers may vary. For example, $f(x) = x$ or $f(x) = x^3$.)

b) Rational $f(x) =$ $\frac{1}{x}$



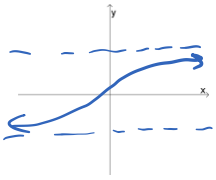
(Answers may vary. For example, $f(x) = \frac{1}{x^2}$.)

c) Trigonometric $f(x) =$ $\sin x$



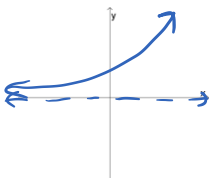
(Answers may vary. For example, $f(x) = \cos x$ or $f(x) = \tan x$.)

d) Inverse Trigonometric $f(x) =$ $\tan^{-1} x$



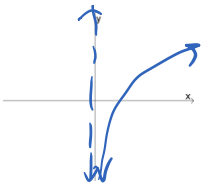
Or $\sin^{-1} x$, $\cos^{-1} x$, $\csc^{-1} x$, $\sec^{-1} x$, $\cot^{-1} x$.

e) Exponential $f(x) =$ e^x



(Answers may vary. For example, $f(x) = 2^x$ or $f(x) = 3^x$.)

f) Logarithmic $f(x) =$ $\ln x$



(Answers may vary. For example, $f(x) = \log_2 x$ or $f(x) = \log_3 x$.)