

The Mean Value Theorem

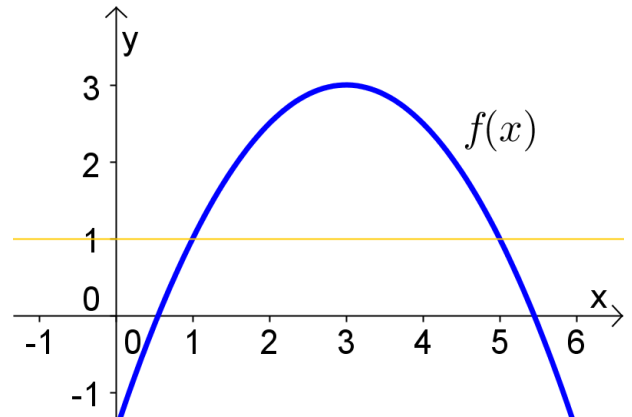
(covers Stewart 4.2)

Rolle's Theorem

Suppose that

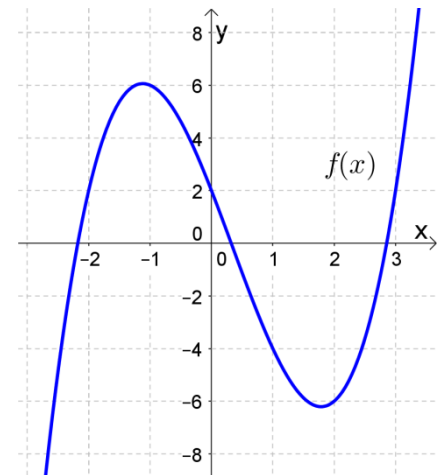
1. $f(x)$ is continuous on $[a, b]$,
2. $f(x)$ is differentiable on (a, b) , and
3. $f(a) = f(b)$.

Then there is at least one number c in (a, b) at which $f'(c) = 0$.



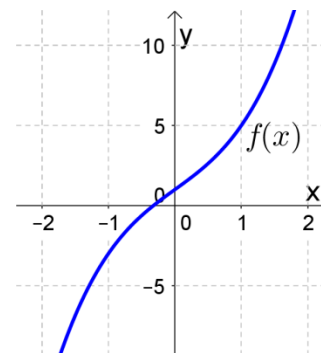
Ex 1.

Verify that $f(x) = x^3 - x^2 - 6x + 2$ satisfies the three hypotheses of Rolle's Theorem on $[0, 3]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.



Ex 2.

Show that the equation $x^3 + 3x + 1 = 0$ has exactly one real solution.

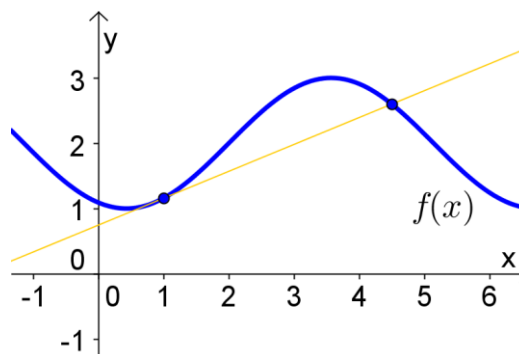
**The Mean Value Theorem**

Suppose that

1. $f(x)$ is continuous on $[a, b]$, and
2. $f(x)$ is differentiable on (a, b) .

Then there is at least one point c in (a, b) at which

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

**Ex 3.**

Verify that $f(x) = \ln(x - 1)$ satisfies the hypotheses of the Mean Value Theorem on $[2, 4]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

